Recovery of Failed Element Signal with a Digitally Beamforming Using Linear Symmetrical Array Antenna

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Abstract:
An antenna array element failure problem is one of the practical and challenging issues in the field of adaptive beamforming. The complete radiation pattern of the array is distorted when any one of the elements fails. Sidelobes level increases, nulls are shifted and null depth is also decreased tremendously. In order to mitigate the problem, we have given a new and easy approach coined as conjugate symmetry approach of the array. In this, the failed element is given a conjugate of the output from its symmetrical counterpart element in the array. The Classical Dolph-Chebyshev and Taylor pattern are taken as the test antenna. The simulation results of both patterns show the validity and presentation of the proposed approach.

Keywords: linear symmetrical array, failure correction, array factor, beamforming
I. INTRODUCTION

Detection and correction of failed element in array antenna is a problem of concern for the researchers. Element failure can degrade the antenna pattern so as to affect the beam steering and null steering. A single element failure gives rise to sidelobes, diminishing of nulls depth, null shifting etc. Several elements failures can almost destroy the pattern. In literature different techniques are available to correct the damage pattern. Recently some papers have addressed a technique to improve the array pattern in the existence of failed elements by optimizing the weights of the active array elements to achieve the pattern as close to the original one [1]. In [2] the failed element signals are reconstructed from its neighboring elements. Both methods are used for array correction and needs a digital beamforming array. Peters et al [3] proposed a conjugate gradient method to reconfigure the amplitude and phase distribution of the active array elements by reducing the average sidelobe levels. Lorenzo et al [4] proposed a time modulated array technique for failure correction. W. P. Keizer and H. Stark, [5] used a large mono-pulse phased array antenna for correction of failed elements using the active amplitude weighting. Levitas et al. [6] proposed a practical method of amplitude and phase compensation using elements near the failed TRM. Lozano et al., J. A. Hejres et al. and Z. U. Khan et al. [7-10] presented compensation as maintaining fixed nulls, null steering and independent null steering in phased array. Chenglong Zhu et al. developed an algorithm for impaired, DOA estimation and diagnosis of faulty array antenna. This technique [11] requires no a priori knowledge of the faulty elements. Once the location of faulty elements has been identified by the fault detection technique [12-16], then we used the technique [17-18] for the failure correction to suppress the sidelobes level in the failed array to achieve the required radiation pattern. Recently O. P. Acharya et al. proposed a method for failure compensation in faulty arrays. The first part of his
study deal with the thinning in the faulty arrays, i.e., to find a limit on the least number of working elements of the array that can recover the desired pattern while the second part deals with the maximum number of faulty elements that can be compensated using particle swarm optimization [19], but this method only reduce the SLL. The symmetrical linear array is of great importance that has already shown useful results to achieve the required SLL and null depth level [20]. The second advantage is that, as we have required half number of damage patterns instead of finding all damage patterns for the detection of faulty elements [21]. The signal reconstruction technique is based on the symmetrical linear array, in which the received signals at neighboring elements only are different by a phase constant.

In this paper, we have given a new and easy approach based on the symmetry of the array. A linear symmetrical array antenna is used for failure correction where the failed elements signal is reconstructed from its symmetrical counterpart element in the array by taking the conjugate. The simulation results show the validity and presentation of the proposed approach.

The rest of the paper is organized as follows. The failure correction problem formulation is discussed in section II, while in section III, the proposed solution is provided. Section IV describes the simulations and the results. Section V is dedicated for conclusions and future work directions.

II. PROBLEM FORMULATION

Consider a linear array of $P$ elements in which all the elements are placed symmetrically along the x-axis. The total number of elements is $P = 2N$. The signal received by the $nth$ element is given by.

$$x_n = A_i \exp \{jk(n kd \sin \theta_i)\} \quad (1)$$
where $A_i$ is the amplitude of the wave whereas $n = \pm 1, \pm 2, \ldots, \pm N$. The spacing between the adjacent elements is $d$, while $\theta$ is the angle from broadside. $k = \frac{2\pi}{\lambda}$ is the wave number with $\lambda$ as wavelength. For an arbitrary linear array, the array factor can be given by [22],

$$AF(\theta_i) = w^H s(\theta_i)$$

(2)

where $w = [w_{-N}, w_{-N+1}, w_{-N+2}, \ldots, w_{-1}, w_1, \ldots, w_{N-2}, w_{N-1}, w_N]^T$ is the weight vector and $s(\theta_i)$ is the steering vector given as follows

$$s(\theta_i) = [\exp(-j\frac{2N-1}{2}kd \sin \theta_i), \exp(j\frac{1}{2}kd \sin \theta_i), \exp(j\frac{1}{2}kd \sin \theta_i), \ldots, \exp(-j\frac{2N-1}{2}kd \sin \theta_i)]^T$$

The output of each element is multiplied by the weights to form the array output signal, we can write the normalized array factor for $N$ number of elements as follows

$$AF(\theta_i) = \sum_{n=1}^{N} w_n \cos \left[ \frac{(2n-1)}{2}kd \sin \theta_i \right]$$

(3)

where $w_n$ are the complex weights of the antenna, selected to steer a desired array pattern and $x_n$ is the signal received at the $n$th element. Now if $m$th element failed in the array, the array factor for the damaged pattern can be given by equation (4) as follows

$$AF(\theta_i) = \sum_{n=1}^{N} \sum_{n \neq m}^{N} w_n \cos \left[ \frac{(2n-1)}{2}kd \sin \theta_i \right]$$

(4)

Due to $m$th element failure the whole radiation pattern disturbs, its sidelobes level (SLL) raises high, nulls are damaged and shifted from their original positions. The power pattern of the original and damaged array is depicted in Fig-1.
Various techniques are available in literature to correct the faulty antenna pattern, however, our proposed technique is simple and fast to achieve the required pattern as close to that of the original array.

III. PROPOSED SOLUTION

In this section, we develop a simple algorithm for the reconstruction of the failed element signal from its symmetrical element. So if any $n$th element fails (zero output) in the array, then we can recover the $n$th failed element signal from its symmetrical element ($-n$) by its conjugate. From Fig.2 it is shown that when $n$th element failed, the switch will be on and the connection will recover the $n$th failed element signal from the symmetrical element by taking its conjugate. Similarly for any failed element the switch is on, and will recover the failed element signal from its symmetrical element by taking its conjugate.

The received signal $x_n$ by the $n$th element is given by the following equation

Figure 1. The original and the 4th element faulty power pattern.
\[ x_n = A_i \exp(j(nkd \sin \theta)) \]  

(5)

where \( A_i \) is a complex constant. If \( n \)th element fails \((x_n = 0)\), then the switch of the symmetrical counterpart \((x_{-n})\) of the \( n \)th element is turned-on to reconstruct the failed \( n \)th element signal by taking its conjugate as shown in Fig.2.

\[ x_n = (x_{-n})^* \]  

(6)

The weights of beamformer at each antenna element to form the array output power pattern are given as follows

\[ y = \sum_n w_n x_n \]

where \( w_n \) are the complex weights of the antenna element chosen to steer a specific array power pattern. The signal at any \( n \)th antenna element is correlated to that at the \( n \)th element by a simple conjugate symmetry relation.

**Figure 2.** The Structure of symmetrical linear array antenna
IV. SIMULATION RESULTS

In the simulation, we consider a Classical Dolph-Chebyshev and Taylor linear array of 30 elements with $\lambda/2$ inter-element spacing is used as the test antenna. The array factor in this case represents a -40 dB constant SLL with the nulls at particular angles. Analytical techniques are used to find out the non-uniform weights for Classical Dolph-Chebyshev and Taylor pattern array. The simulation results show the performance of the proposed technique in the presence of element failure. In case of element failure, the failed element signal is reconstructed from the symmetrical element by taking its conjugate.

Case A: At the first instant, we consider a Classical Dolph-Chebyshev array, and we assumed that element number four becomes failed, due to $4^{th}$ element failure ($w_4$), the whole pattern of the array becomes disturbed, its SLL raises high, nulls are damaged and shifted from their original positions. Due to ($w_4$) element failure, the SLL raises to -32.1 dB. The advantage of applying the proposed approach, we received the same pattern as that of the original one. In Fig. 3, the solid line is the original array with weight excitation set to -40 dB Chebyshev pattern. The dash line is the array with $4^{th}$ element ($w_4$) assumed to be failed. After using the proposed technique, the dotted line is the corrected pattern. By the proposed technique, with the recovery of SLL we achieved the nulls back at their required positions as that of original array.
Figure 3. The original pattern, the 4th element faulty and corrected pattern.

Now we assumed that the 3rd \((w_3)\) and 6th \((w_6)\) elements become damaged in the array. Due to \(w_3\) and \(w_6\) elements failures, the SLL are raises to -26.86 dB and nulls are completely damaged and loses its depth. After applying the proposed method, we get the desired SLL and nulls back at their original positions. In Fig. 4 the solid line indicates the original pattern, the dashed line represents \(w_3\) and \(w_6\) element failure in the array. From Fig. 4 it is clear that as the number of faulty elements increase, its SLL increases and nulls are damaged. After using the proposed method, the dotted line is obtained which is the corrected pattern as that of the original one.
Now we discuss the failure of 1\textsuperscript{st}, 3\textsuperscript{rd} and 10\textsuperscript{th} element in an array of 30 elements. From Figure 4 it is clear that SLL are raises very high, nulls are badly damaged and loses its depth. Due to these element failure, its SLL are raises to -23.26 dB and nulls are shifted from their original positions. In Fig. 5, the solid line represent the original pattern. The dashed line indicates the same array with 1\textsuperscript{st}, 3\textsuperscript{rd} and 10\textsuperscript{th} elements failed. After using the proposed correction scheme, the dotted line is obtained, which represents the corrected pattern and is nearly equal to the original one. After using the proposed method, the corrected pattern nearly overlaps the original pattern in any desired direction.
Figure 5. The original, 1st, 3rd and 10th \( w_1, w_3, \text{and} w_{10} \) elements failure and corrected pattern.

Again we consider the same array of 30 elements, but this time, the failure of elements is assumed on the left of the array center. In this we assumed to fail \( w_{-1}, w_{-3} \text{ and } w_{-10} \) elements. The failures of \( w_{-1}, w_{-3} \text{ and } w_{-10} \) means that elements 1st, 3rd and 10th are failed on the left of the array. Fig. 6 shows the original, damaged and corrected pattern for the \( w_{-1}, w_{-3} \text{ and } w_{-10} \) elements. The failure of \( w_{1}, w_{3} \text{ and } w_{10} \) elements on the right of the array centre gives the same pattern as that of the failure of \( w_{-1}, w_{-3} \text{ and } w_{-10} \), elements to the left of the array centre. The solid line indicates the original array pattern, dashed line is the damaged pattern and the dotted line is the corrected pattern. The symmetrical linear array has the advantage, that for both types of failures we received the same pattern, due to which the computational complexity is reduced to half [21].
Figure 6. The original pattern, the $w_{-1}$, $w_{-3}$ and $w_{-10}$ elements failure and corrected pattern.

Case B: In this case we assumed that the 2nd ($w_2$) and the 3rd ($w_3$) elements in the array becomes fail and the desired user changes their position, then the main beam can be steered in the desired direction. The main beam can be directed at any angle in the desired direction. The power pattern for the main beam steering $\theta_s$ is given by the following expression as follows

$$AF(\theta) = \sum_{n=-M}^{M} w_n \exp jnk(\cos \theta_i - \cos \theta_s)$$

where $\theta_s$ is the main beam to which it can steered in the desired directions. Fig. 7 shows the main beam pointing at $\theta_s = 115^\circ$. 

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Figure 7. The original, the 2\textsuperscript{nd} and the 9\textsuperscript{th} element failure and the corrected pattern.

Case C: In this case, if element failures occurred on the left as well as on the right of the array center. Now we assumed that elements \( w_{-8}, w_{-2}, w_4 \) \textit{and} \( w_6 \) are failed. From Fig. 8 it is clear that due to these four element failures, the SLL and nulls are badly damaged. The solid line indicates the original array obtained by Chebyshev pattern. The dashed line is the damaged pattern with element failure at positions \( w_{-8}, w_{-2}, w_4 \) \textit{and} \( w_6 \). From Fig. 8 it is clear that without using the proposed technique, one cannot recover the desired pattern. The recovery of SLL for this case is impossible. The desired communication becomes a dream with such type of failure in the array antenna. The only possibility of recovery the desired SLL and null positions at their required positions is using to recover the failed elements signals from its symmetrical elements by taking conjugate.
Figure 8. The original pattern, and the $w_{-8}, w_{-2}, w_4$ and $w_6$ elements failure pattern.

**Case D:** In this case, a Taylor linear array of 30 elements with a SLL of -40 dB with $\bar{n} = 6$ is considered. Analytical techniques are used to find out the non-uniform weights for Taylor array. Fig. 9 shows the result of the proposed method from the data received from an array of 30 elements. In Fig. 9, the solid line represent the original pattern with the array weights set to -40 dB Taylor taper with $\bar{n} = 6$. The dash line is the same array with 4th ($w_4$) element fails. Due to 4th element failure, the SLL increase and nulls are damaged and displaced from their original position. Our objective is to reduce the SLL and placed the nulls at their original positions. After applying the proposed technique, the dotted line is obtained which shows the required pattern as that of original one.
In this case, the failure in the array antenna occurs at the $3^{rd}$ ($w_3$) and $4^{th}$ ($w_4$) position. Due to elements failure, the whole radiation pattern disturb in terms of SLL and null depth, and for the recovery of the required signal we proposed a method in which the $3^{rd}$ and $4^{th}$ elements signals are recovered from its symmetrical counterpart by taking its conjugate. In Fig. 10, the solid line indicates the original pattern, the dashed line represent $3^{rd}$ and $6^{th}$ element failure in the array. From Fig. 10 it is clear that as the number of faulty elements increase, its SLL increases and nulls are damaged. After using the proposed method, the dotted line is the corrected pattern.
Figure 10. The original pattern, the 3rd and 6th elements failure and corrected pattern.

Fig. 11 shows the array pattern of 30 elements with -40 dB SLL, the solid line represent the original pattern. The dashed line indicates the same array with 1st, 3rd and 10th elements failed. After using the propose method, the dotted line is obtained, that is the required pattern which nearly overlaps the original pattern in any desired direction. Again we consider the same case, but this time, the failure of elements is assumed on the left of the array center. In this case, we assumed to fails 1, 3 and 10 elements. The failures of 1, 3 and 10 means that elements 1st, 3rd and 10th are failed on the left of the array. Fig. 12 shows the original, damage and corrected pattern for 1, 3 and 10 elements. The failure of 1, 3 and 10 on the right of the array centre gives the same pattern as that of the failure of 1, 3 and 10 to the left of the array centre. The solid line indicates the original array pattern, dashed line is the damaged pattern and the dotted line is the corrected pattern. The symmetrical linear array has the
advantage, that for both types of failures we received the same pattern, due to which the computational complexity is reduced to half [20].

**Figure 11.** The original pattern, the $w_1, w_3$ and $w_{10}$ elements failure and corrected pattern.

**Figure 12.** The original pattern, the $w_{-1}, w_{-3}$ and $w_{-10}$ elements failure and corrected pattern.
**Case E:** - The performance of the proposed method is compared with the conventional method [19] and [20]. The parameters of damaged pattern are disturbed in terms of SLL, null depth level (NDL) and displacement of nulls from their original positions. In [19] the main goal is to restore the SLL to its original pattern by adjusting the weights of the remaining elements but the NDL and positions of nulls back to their original positions is still an issue to be taken in to account. Our proposed method, resolve the issues of SLL, NDL and position of nulls to their original positions. The parameters of the damaged, proposed and conventional [19] are given in Table I.

<table>
<thead>
<tr>
<th>Failed position</th>
<th>SLL</th>
<th>NDL</th>
<th>SLL</th>
<th>NDL</th>
<th>SLL</th>
<th>NDL</th>
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</thead>
<tbody>
<tr>
<td>5</td>
<td>-31.15</td>
<td>-63.71</td>
<td>-40.01</td>
<td>-105.79</td>
<td>-39.51</td>
<td>-78.49</td>
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<tr>
<td>4,7</td>
<td>-25.67</td>
<td>-52.53</td>
<td>-40.01</td>
<td>-108.76</td>
<td>-39.65</td>
<td>-47.53</td>
</tr>
</tbody>
</table>

From Fig. 13 it is clear that [19] only resolve the issue of SLL but the proposed method overlaps the original pattern and we get the same pattern as that of the original array.

**Figure 13.** The original, 5th element damaged, proposed, and conventional method pattern.
Now if the desired user changes their position, then we can steer the main beam in the direction of target. In this case, the main beam is pointing at an angle of $\theta_s = 120^\circ$ with the nulls recovered at their required positions.

**Figure 14.** The original, 5th element damaged pattern, and main beam pointing at an angle $120^\circ$.

**Figure 15.** The original, 2nd element damaged, proposed and conventional method pattern.
**Case F:** In this case, the performance of the proposed method is compared with [20]. In [20] the issue of NDL and position of nulls back at their original positions is solved but we achieved higher SLL. From Fig.15 it is clear that with conventional method [20] we get higher SLL espatially the first three picks near the main beam. After applying the proposed method, we nearly get the pattern in terms of SLL, NDL and position of nulls at their original positions as that of the original array. In Fig. 16, the main beam is pointing at an angle of $\theta_s = 120^\circ$ along with the recovered nulls at their original positions. The parameters for the damaged, proposed and conventional method [20] are given in Table II.

### TABLE II Comparison analysis for different number of element failure in an array

<table>
<thead>
<tr>
<th>Failed position</th>
<th>Proposed Method</th>
<th>Convention Method [20]</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>SLL</td>
<td>NDL</td>
</tr>
<tr>
<td>2</td>
<td>-35.15</td>
<td>-74.89</td>
</tr>
<tr>
<td>3,6</td>
<td>-26.45</td>
<td>-68.8</td>
</tr>
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</table>

**Figure 16.** The original, damaged pattern and main beam pointing at an angle $\theta_s = 120^\circ$. 
V. CONCLUSION AND FUTURE WORK

We have proposed a simple technique for failure correction based on symmetric linear array. An antenna element failure problem with an array is one of the practical and challenging issues in the field of adaptive beamforming. Most of the digital beamforming arrays are installed outside the system where the element failure can occur due to several reasons. The complete radiation pattern of the array is distorted when any one of the element is failed. In order to diminish the problem, we have given a new and simple technique based on the symmetry of the array. A linear symmetrical array antenna is used for failure correction where the failed elements signal is reconstructed from the symmetrical element in the array by taking the conjugate. If the elements failed near the centre of the array, the desired communication becomes a dream and we are not able to recover the desired pattern by any optimization technique. Different simulation results are performed for Chebyshev and Taylor patterns. But by using the proposed technique we can achieve the required pattern. This method can be extended to planar arrays.

REFERENCES


