Design and Implementation of a Multiple-Choice E-voting Scheme on Mobile System using Novel $t$-out-of-$n$ Oblivious Signature

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Blind signature schemes allow a user to decide any message without disclosing any information about the message to the signer; while oblivious signature schemes allow a signee to select one of several predetermined messages without revealing any information about the selected message, making such schemes well suited for electronic voting applications. However, the oblivious signature scheme only allows users to select one of the $n$ candidates. In this paper, we first propose a $t$-out-of-$n$ oblivious signature scheme based on the oblivious transfer method to satisfy the security requirements of not only completeness, unforgeability, privacy, but also selection restriction and non-reduplication; making such scheme well suited for multiple-choice e-voting applications. Moreover, we propose multiple-choice e-voting scheme based on the proposed $t$-out-of-$n$ oblivious signature scheme, and implement the scheme in mobile phones to allow users voting securely and conveniently. Security analysis and comparisons of computation and communication efficiency are also provided to validate the proposed schemes.

Keywords: Electronic voting, mobility, blind signature, oblivious signature, security

1. INTRODUCTION

The rapid evolution of network transactions has significantly increased the number of consumers using internet auctions and banking. Protecting user privacy in such applications requires network security technologies, such as network transaction, internet auction, and digital signatures [1-2], which feature several key properties not available with analog signatures, including completeness, unforgeability, undeniability and verifiability. Using public-key cryptography, a signer could sign a message with his exclusive private key. Afterward any verifier can validate the correctness of the signature by using the signer’s public key. Thus unlike a traditional signature, a digital signature cannot be forged, not can the signer deny any signature produced by him or her. Digital signatures can thus be treated as authentic validation by the signer, but can be transferred electronically. This technique is very useful in terms of signer authentication, product validation, data integrity assurance and so on.

However, certain situations require protecting the privacy of signature recipients. In 1982, Chaum [3] introduced a blind signature scheme which satisfies this requirement through introducing the property of blindness. In the scheme, a signee could receive a
signed message without revealing any information about the message. Later schemes [4], [5] build on this concept for use in applications including electric payment systems and secure voting systems which require shielding the potentially sensitive content of the requested messages. Mambo et al. [7]–[9] also proposed a new blind signature combined with a proxy signature [10-11] which can be applied to digital voting systems.

Additionally, Chen [12] first proposed the concept of oblivious signatures, a signature scheme that allows a signee to choose one of a set of predetermined messages for signing without the content of the message to the signee. Oblivious signatures and blind signatures both have the property of “signers cannot know the message they sign from signees,” but oblivious signatures have one more property, they guarantees the signed message is actually belongs to the predefined set of messages, and any message not belonging to this set will be rejected.

However, Tso et al. [13] pointed out that Chen’s proposal [12] did not specifically formalize the scheme or its security properties. As a result they proposed a 1-out-of-\(n\) oblivious signature based on Schnorr’s scheme [14], and provided formal definitions and security requirements of the oblivious signature scheme including completeness, unforgeability and privacy, making oblivious signatures very well suited for electronic voting applications.

Prior to the development of the oblivious signature concept, Rabin introduced the concept of oblivious transfer [15], a protocol in which the sender sends some subsets of some messages, but does not know which messages the recipient has received. In this way the recipient could obtain the desired message without revealing his preference to the sender, and without the recipient being aware of the other message options. In 2004, Tzeng 16 proposed a 1-out-of-\(n\) oblivious transfer and, in 2014 Hao et al. [17] proposed an oblivious transfer scheme based on wireless channel characteristics.

Recently, electronic voting systems are discussed and concerned frequently. More and more countries start to put electronic voting into practice to replace traditional paper voting. For example, in 2005, Estonian announced Estonian E-Voting Laws to adopt electronic voting systems generally. In early days, the digital e-voting systems are usually built based on blind signatures. Some researchers further proposed some signature schemes [6] that combine blind signatures and proxy signatures and applied them to electronic voting systems. However, the application on e-voting systems from blind signatures makes signer unable to know whether the signed message is chosen from one of valid candidates.

Using oblivious signature in e-voting can avoid this kind of problem. Song et al. 18 proposed an electronic voting system based on Tso et al.’s oblivious signature [13]. However, their system increases the loading of voters. Moreover, there is a security problem that attackers can obtain tally result before counting ballots.

In this paper, we first propose a \(t\)-out-of-\(n\) oblivious signature scheme based on oblivious transfer [16], [17] and satisfy the security requirements of completeness, unforgeability, privacy [13], selection restriction and non-reduplication. Security analysis and comparisons of computation and communication efficiency validate the relative effectiveness of the proposed scheme. The proposed scheme is applied to mobile e-voting system, which is implemented in mobile phones that allow users to vote efficiently, securely and conveniently.
The remainder of this paper is structured as follows: Section 1 provides a brief introduction. Section 2 integrates the relevant literature into a framework for analytical discussion. Section 3 outlines the framework for the proposed t-out-of-n oblivious signature scheme and provides its security analysis and performance comparison. The security analysis assesses the capability of the proposed framework to ensure completeness, unforgeability, privacy, selection restriction and non-reduplication. This paper compares the effectiveness of the proposed oblivious signature scheme with other similar schemes in terms of computing requirements, transmission loading, and scheme properties. The details for the mobile multiple-choice e-voting application from the proposed t-out-of-n oblivious signature scheme are presented in Section 4 and our implementation is described in Section 5. Concluding remarks are offered in Section 6.

2. RELATED WORKS

This section introduces the concept and protocols of the proposed signature scheme.

2.1 Oblivious Signature

In 2008, Tso et al. [13] proposed a fair game example to illustrate the operations of the oblivious signature scheme (see Fig. 1). Assume one operator and multiple players. First, one player plays rock, paper scissors (assume he chooses ‘rock’). The parameters corresponding to the selected item (i.e., ‘rock’) are used to calculate $s$ which is then transmitted to the operator, but the operator is unable to determine the player’s selection based on $s$. After confirming the player’s identity, the operator uses $s$ to calculate values for rock, paper or scissors, and then signs the calculated values. Based on this determination, the signer transmits his/her signature to the player who can only receive the signature for the item ‘rock’. Finally, after the operator announces the solution, the operator determines the winner, and the winner uses the operator’s signature to claim his/her prize.
2.2 Chen’s Oblivious Signature Agreement

Chen’s 1994 oblivious signature concept [12] proposed two types of oblivious signature agreements. The first type uses $n$ keys. In this agreement, the members are $n$ signers (or one signer with $n$ keys) and one or more recipients. The following three characteristics must be met:

1. By implementing this agreement, the recipient can obtain the signature value of a message, and this signature value uses $n$ to choose a signature key according to the recipient’s selection.
2. Even with possession of this signature key, the signer is unable to determine which is the signing key from the signature value.
3. In the event of a dispute, others are unable to determine the signature key based on the recipient’s signature value.

The second type uses $n$ messages. In this agreement, the members are one signer and one recipient. Assuming both know $n$ messages, the following three conditions must be met:

1. By implementing this agreement, the recipient can only select a message from $n$ for signing.
2. The signer is unable to determine which message the recipient has selected.
3. In the event of a dispute, others are unable to determine which message corresponds to the recipient’s signature value.

3. PROPOSED SCHEME OF NOVEL T-OUT-OF-N OBLIVIOUS SIGNATURE

This section provides a complete introduction to the operational process of the proposed $t$-out-of-$n$ oblivious signature agreement, and its discussion including security analysis and performance comparison. This agreement allows the recipient to select $t$ messages from $n$ for the signer to sign. If the recipient selects a message which does not belong to $n$, then the recipient will finally receive a notification from the verifier that the signer cannot be verified. Through this process, the signer is unable to determine which message was selected by the recipient. Table 1 illustrates the notations used in the protocol.

3.1 Attacker model

In our scheme, we assume the channels between the signer and the recipient, the recipient and the verifier, and the signer and the verifier are insecure. Any identity (i.e. the signer, the recipient, or the verifier) communicates with another via an insecure public channel, offering adversaries opportunities to intercept. In the following, we present the assumptions of the attacker model [19, 20].
### Table 1 Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>$(e, N)$</td>
<td>RSA public key of signer</td>
</tr>
<tr>
<td>$(d, N)$</td>
<td>RSA private key of signer</td>
</tr>
<tr>
<td>$Z_N$</td>
<td>complete system of residues modulo $N$</td>
</tr>
<tr>
<td>$Z'_N$</td>
<td>reduced set of residues modulo $N$</td>
</tr>
<tr>
<td>$n$</td>
<td>The number of messages</td>
</tr>
<tr>
<td>$t$</td>
<td>The number of chosen messages</td>
</tr>
<tr>
<td>$m_i$</td>
<td>The $i$th message</td>
</tr>
<tr>
<td>$a_j$</td>
<td>The value of the subscript of the selected message $m_j$, $j \in (1, t)$</td>
</tr>
<tr>
<td>$\sigma(x)$</td>
<td>The signature value of $x$</td>
</tr>
<tr>
<td>$H(\cdot)$</td>
<td>A public cryptographic one-way hash function</td>
</tr>
</tbody>
</table>

(1) An adversary may eavesdrop on all communications between protocol actors over the public channel.

(2) An attacker can modify, delete, resend and reroute the eavesdropped message.

(3) An attacker cannot be a legitimate signer.

(4) The attacker knows the protocol description, which means the protocol is public.

### 3.2 Security Requirements

This agreement can be applied more flexibly to meet Tso’s three security criteria (completeness, unforgeability and privacy) and two more properties (selection restriction and non-reduplication.) The security requirements for the proposed $t$-out-of-$n$ oblivious signature scheme are defined as follows.

**Definition 1 (Security requirements of the proposed $t$-out-of-$n$ oblivious signature scheme)** The proposed $t$-out-of-$n$ oblivious signature scheme is secure if it achieves (1) Completeness, (2) Unforgeability, (3) Privacy of selected messages, (4) Selection restriction, and (5) Non-reduplication.

The security requirements of our scheme are listed as follows:

(1) **Completeness**: As long as the recipient and the signer can implement the agreement honestly, once the agreement is completed the recipient can obtain the signed message.

(2) **Unforgeability**: Despite the algorithm being publicly published, attackers still have difficulty creating a forged signature within an acceptable time frame.

(3) **Privacy of selected messages**: The signer is unable to determine the recipient’s selection.

(4) **Selection restriction**: The recipient is unable to get a valid signature of any message except the $n$ messages.
Select $r_i \in \mathbb{Z}_N^*$, $i = 1, 2, ..., n$

Calculate $s_i = H(m_i, r_i)^{e_i} \mod N$

Verify $H(m_i, r_i)^{e_i} = s_i^{e_i} \mod N$

If this is correct, select $b_j \in \mathbb{Z}_N^*$, $j = 1, 2, ..., t$ and $t < n$,

calculate $c_j = b_j^{e_i} \mod N$, $1 \leq a_i, a_j, ..., a_t \leq n$

Calculate $\beta_j = c_j \mod N$

Select $\sigma(m_j) = (s_i, v_j)$

Fig. 2. Signing phase process

(5) Non-reduplication: The recipient cannot get more than one signature on the same message in a signing process.

3.3 Protocol of Proposed $t$-out-of-$n$ Oblivious Signature Scheme

A complete introduction of the process is provided below, including the roles of the three participants (signer, recipient and verifier) and the three phases (initiation, signing and verification).

(1) System initiation phase

This phase first defines parameters and has the signer select an appropriate hash function $H$ and generates the required public key $(e, N)$ and private key $(d, N)$ as follows:

Step 1: Select two large prime numbers $p, q$

Step 2: Calculate $N = p \times q$

Step 3: Calculate $\phi(N) = (p - 1)(q - 1)$

Step 4: Select $e \ni \text{GCD}(e, \phi(N)) = 1$

Step 5: Calculate $d \ni ed = 1 \mod \phi(N)$

(2) Signing phase

This phase explains how the recipient obtains the signer’s complete signature. The process is illustrated in Fig. 2.

Step 1: The signer first randomly selects $i$ variables $r_i$, $i = 1, 2, ..., n$, and calculates $s_i = H(m_i, r_i)^{e_i} \mod N$, and then transmits $(s_i, r_i, m_i)$, $i = 1, 2, ..., n$ to the recipient.

Step 2: The recipient receives $(s_i, r_i, m_i)$ and then verifies $H(m_i, r_i)^{e_i} = s_i^{e_i} \mod N$. If it is completely correct, the recipient selects $t$ messages from $n$ and $t$ variables $r_{a_j}$ which correspond to the selected $t$ messages, where $j = 1, 2, ..., t$ and $t < n$, $1 \leq a_i, a_j, ..., a_t \leq n$. The recipient then selects a random variable $b_j$, calculates $c_j = b_j^{e_i} \mod N$ and transmits $c_j$, $j = 1, 2, ..., t$ to the signer.
Step 3: The signer receives $c_j$ and calculates $\beta_j = c_j^d \mod N$, before transmitting $\beta_j, j=1,2,...,t$ to the recipient.

Step 4: The recipient receives $\beta_j$, then uses the inverse of $b_j$ to calculate $v_j = b_j^{-1}\beta_j \mod N$ to obtain the complete signature $\sigma(m_u) = (s_u, v_j)$ (where $\sigma(x)$ is the signature value of $x$).

(3) Verification phase

This phase verifies that the signature received by the recipient is correct. The details are illustrated in Fig. 3.

Step 1: The recipient transmits $(\sigma(m_u), m_u)$ to the verifier.

Step 2: The verifier checks whether $s_u = H(m_u, v_j) \mod N$ is correct. If so, then the signature is verified.

3.4 Security Analysis

This section assesses the security of the proposed method in terms of completeness, unforgeability, privacy, selection restriction and non-reduplication:

(1) Completeness

In Step 2 of the signing phase, the recipient verifies $s_u = H(m_u, v_j) \mod N$ to determine the authenticity of the signature, we have $H(m_u, v_j) = H(m_u, (b_j^{-1} \beta_j)^{e_j}) = H(m_u, (b_j^{-1} c_j^{d_j})^{e_j}) = H(m_u, (b_j^{-1} c_j^{d_j})^{e_j}) = H(m_u, (b_j^{-1} r_j)^{e_j}) = H(m_u, r_u) = s_u \mod N$. Therefore the completeness of the oblivious signature $(s_u, v_j)$ is proven.

(2) Unforgeability

The security of unforgeability can be proved via Def. 2, Def. 3, Thm. 1, and Thm. 2.

Definition 2 (RSA problem) Let $(e', N')$ be RSA public keys and $c' = m'' \mod N'$, where $m', c' \in Z_q$. If $m'$ can be evaluated from given $e', N'$ and $c'$, then we say RSA problem can be solved. (The probability of solving this problem is denoted as $\Pr(m' | e', N', c') = \epsilon_{rsa}$.)

Theorem 1 (Unforgeability) In our protocol, if a recipient can forge the signer’s signature, then the RSA problem can be solved.

Proof. A recipient R tries to forge the signer’s signature by evaluating $\beta_j$ from given $c_j, e, N$. Let $RO_1$ be a random oracle: input $c_j, e$ and $N$ to output $\beta_j$ such that
Let \( \beta^* = c_j \mod N \). (i.e. \( RO_i(c_j, e, N) \Rightarrow \beta_j^* : \beta^* = c_j \mod N \)). In definition 2, Let \( c_j \leftarrow c^* \), \( e \leftarrow e^* \) and \( N \leftarrow N^* \) be input parameters of \( RO_i \) and obtain output \( \beta_j^* \).

Let \( m_i^* \leftarrow \beta_j^* \), then \( m_i^* \) is evaluated. Therefore, \( \Pr(\beta_j^* | c_j, e, N) \leq \Pr(m_i^* | e^*, N^*, c^*), \) which means the RSA problem can be solved if \( RO_i \) exists.

**Definition 3 (RSA problem under known plaintext attack)** Let \( (e^*, N^*) \) be RSA public keys and \( c_i^* = m_i^* \mod N^* \), where \( m_i^*, c_i^* \in \mathbb{Z}_{N^*}^* \) and \( i = 1, 2, ..., n + 1 \). If \( m_i^* \) can be evaluated from given \( e^*, N^*, (m_i^*, c_i^*), c_{i+1}^* \), \( i = 1, 2, ..., n \), then we say RSA problem under known plaintext attack can be solved. (The probability of solving this problem is denoted as \( \Pr(m_i^* | e^*, N^*, (m_i^*, c_i^*), c_{i+1}^*), e_{rsa} = \epsilon \).

**Theorem 2 (Unforgeability under replay attack)** In our protocol, if a recipient can forge \( \beta_{i+1}^* \) from given \( c_{i+1}^* \) and \( n \) pairs of \( (c_i^*, \beta_i) \), \( i = 1, 2, ..., n \), then the RSA problem under known plaintext attack can be solved.

**Proof.** A recipient \( R \) tries to forge the signer’s signature by evaluating \( \beta_{i+1}^* \) from given \( c_{i+1}^* \) and \( n \) pairs of \( (c_i^*, \beta_i) \). Let \( RO_2 \) be a random oracle: input \( c_{i+1}^* \) and \( n \) pair of \( (c_i^*, \beta_i) \) to output \( \beta_{i+1}^* \) such that \( \beta_j^* = c_j \mod N \), \( i = 1, 2, ..., n + 1 \). (i.e. \( RO_2(c_{i+1}^*, (c_i^*, \beta_i)) \Rightarrow \beta_{i+1}^* : \beta_j^* = c_j \mod N \)). In definition 3, Let \( c_{i+1}^* \leftarrow c_{i+1}^* \), \( c_i^* \leftarrow c_i^* \), \( \beta_i \leftarrow m_i^* \), \( e \leftarrow e^* \) and \( N \leftarrow N^* \) be input parameters of \( RO_2 \) and obtain output \( \beta_{i+1}^* \). Let \( m_{i+1}^* \leftarrow \beta_{i+1}^* \), then \( m_{i+1}^* \) is evaluated. Therefore, \( \Pr(\beta_{i+1}^* | e, N, (\beta_i, c_i, \beta_{i+1}), c_{i+1}^*), e_{rsa} \leq \Pr(m_{i+1}^* | e^*, N^*, (m_i^*, c_i^*), c_{i+1}^*), e_{rsa} \), which means the RSA problem under known plaintext attack can be solved if \( RO_i \) exists.

(3) **Privacy of selected message**

In Step 2 of the signing phase, the recipient randomly selects a blind factor \( b_j \) to blind \( c_j = b_j r_j \mod N \) where \( c_j \) is transmitted to the signer. If an attacker intercepts \( c_j \), it is obvious that he/she cannot determine \( b_j \) or \( r_j \) from \( c_j \). As for the signer, he/she can decrypt \( b_j \) by calculating \( b_j = c_j^d r_j^{-d} \mod N \), as a result, he/she will obtain \( n \) potential \( b_j \) corresponding to \( n \) random numbers \( r_j \), since he/she doesn’t know which \( b_j \) is picked by the recipient, the probability \( r_j \) is selected by the recipient is still \( 1/n \).

Thus the recipient’s privacy is perfectly protected.

(4) **Selection restriction**

In Step 2 of the signing phase, the recipient makes his/her selections by choosing \( t \) random numbers \( r_j \) and generating \( c_j \). If he/she picks a number \( r’ \) which is not belong to \( r_j, i = 1, 2, ..., n \), he/she will receive \( \beta = (b_j r^d) \mod N \) from the signer and the extracted \( v_j = b_j^{-1} \beta_j = (r^d) \mod N \) which will not pass the examining equation.
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\[ s'_{ij} = H(m_i, v_j') \mod N \quad (\because r' \neq r) \]. Moreover, if a recipient attempts to get a signature on an irrelevant message \( m' \), since the final signature is composed of \( \sigma(m'_j) = (s'_j, v'_j) \), where \( s_j = H(m_j, r_j)^e \mod N \) is produced by the signer and bound with the message \( m_j \), the recipient cannot choose a message besides the predetermined messages unless he/she can find the numbers \( (s', v') \) that satisfy \( (s')^e = H(m', v') \mod N \), which contradicts the unforgeability property.

The security of selection restriction can be proved via Def. 4, Def. 5, Thm. 3, and Thm. 4.

**Definition 4 (Modified RSA signature forgery problem)** Let \((e, N)\) be RSA public keys and \( s' = H(m'_i, m'_j) \mod N \). If \((m'_i, s')\) can be evaluated from given \((m_i, e, N)\), then we say modified RSA signature forgery problem can be solved. (The probability of solving this problem is denoted as \( \Pr(m'_i, s' | m_i, e, N) = \epsilon_{mrsasf} \)).

**Theorem 3 (Selection restriction)** In our protocol, if a recipient can evaluate a signature on an irrelevant message, then the modified RSA signature forgery problem can be solved.

**Proof.** A recipient \( R \) tries to evaluate a signature on an irrelevant message by evaluating \((s', v')\) from given \( m' \). Let \( RO \) be a random oracle: input \((m', e_0, N_0)\) to output \((s', v')\) such that \( s'^e = H(m'^e, v'^e) \mod N_0 \). (i.e. \( RO(m', e_0, N_0) \Rightarrow (s', v') : s'^e = H(m'^e, v'^e) \mod N_0 \).) In definition 4, Let \( m' \leftarrow m_i \), \( e_0 \leftarrow e \) and \( N \leftarrow N \) be input parameters of \( RO \) and obtain output \((s', v')\). Let \( s \leftarrow s' \), \( m_j \leftarrow v' \), then \((m_j, s)\) is evaluated. Therefore, \( \Pr(s', v' | m'_i, e_0, N_0) \leq \Pr(m'_i, s' | m_i, e, N) = \epsilon_{mrsasf} \), which means the modified RSA signature forgery problem can be solved if \( RO \) exists.

**Definition 5 (Modified RSA signature forgery problem under known plaintext attack)** Let \((e, N)\) be RSA public keys and \( s'_i = H(k, m'_j) \mod N \), where \( k, m, s \in Z_n \) and \( i = 1, 2, \ldots, n+1 \). If \((m_{s+1}, s_{s+1})\) can be evaluated from given \( e, N, k_{s+1} \) and \((k, t_i, s_i)\), where \( t_i = m'_i \), \( i = 1, 2, \ldots, n \), then we say modified RSA signature forgery problem under known plaintext attack can be solved. (The probability of solving this problem is denoted as \( \Pr(m'_i, s'_{s+1} | e, N, k_{s+1} = \epsilon_{mrsasf_{kpa}} \)).

**Theorem 4 (Selection restriction under replay attack)** In our protocol, if a recipient can evaluate a signature on an irrelevant message from given \( n \) triples \((m_i, s_i, r_i)\), where \( r_i = v'_i \), \( i = 1, 2, \ldots, n \), then the modified RSA signature forgery problem under known plaintext attack can be solved.

**Proof.** A recipient \( R \) tries to evaluate a signature by evaluating \((s'_{s+1}, v'_{s+1})\) from given
Let $m_{n+1}', e_0, N_0$ and $n$ triples $(m_i, s_i, r_i)$ to output $(s_i^{n_0}, v_{n+1})$ such that
\[ s_i^{n_0} = H(m_i, v_{n+1}) \mod N_0, \]
where \( r_i' = v_i^{n_0}, \ i = 1, 2, \ldots, n+1 \). (i.e. $RO_4(m_{n+1}', e_0, N_0, (m_i, s_i, r_i)) \Rightarrow (s_i^{n_0}, v_{n+1}) : s_i^{n_0} = H(m_i, v_{n+1}) \mod N_0$.) In definition 5, Let $e_0 \leftarrow e$, $m_{n+1}' \leftarrow k_{n+1}$, $N_0 \leftarrow N$ and $(m_i, s_i, r_i) \leftarrow (k, s_i, t_i)$ be input parameters of $RO_4$ and obtain output $(s_i^{n_0}, v_{n+1})$. Let $s_{n+1} \leftarrow s_{n+1}$ and $m_{n+1} \leftarrow v_{n+1}$, then $(m_{n+1}, s_{n+1})$ is evaluated. Therefore, \[ \Pr(s_{n+1}, v_{n+1} | m_{n+1}', e_0, N_0, (m_i, s_i, r_i)) \leq \Pr(m_{n+1}, s_{n+1} | e, N, k_{n+1}, (k, t_i, s_i)) = \varepsilon_{\text{normal}}. \]
which means the modified RSA signature forgery problem under known plaintext attack can be solved if $RO_4$ exists.

(5) Non-reduplication

If a recipient tries to get two signatures on the same message, he/she can randomly choose $b_1, b_2$ in the signing phase and calculate $c_1 = b_1 r_{r_i} \mod N$, $c_2 = b_2 r_{r_i} \mod N$. However, after extracting the signature parameters $v_1, v_2$, it turns out
\[ v_1 = b_1^{-1} \beta_1 = v_2 = b_2^{-1} \beta_2 = r_i^{n_0} \mod N, \]
the recipient still gets the same signature on the same message, this prevents recipients from getting more than one signature on the same message. Notice that if a normal 1-out-of-$n$ oblivious signature scheme wants to achieve multiple-choice functionality, it will need to repeat the signing phase $t$ times, meanwhile this cannot accomplish the non-reduplication property.

3.5 Performance Comparison

This section provides a performance comparison for the proposed oblivious signature agreement in terms of computation loading, transmission loading and transmission frequency. Such a comparison requires first establishing a reference point in the algorithm. For example, the formulae and algorithm are first run prior to exponentiation, e.g., the algorithm for formula $x^{ab}$ is first run to calculate $ab$ before exponentiation, and thus counting for a single exponentiation instance. In addition, “division” can be considered an instance of “exponentiation” because division is a multiplication of inverse, and the inverse calculation uses a Euclidian approach. We then compare the computation loading of the 1-out-of-$n$ oblivious signature scheme, as shown in Table 2, where $T_E$ represents the time required for modular exponentiation.
Table 2 1-out-of-n Computation Loading Comparison

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<tr>
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<tbody>
<tr>
<td>Recipient (R)</td>
<td>(2n+10)TE</td>
<td>(2n+2)TE</td>
<td>(n+2)TE</td>
</tr>
<tr>
<td>Signer (S)</td>
<td>(3n)TE</td>
<td>(2n+1)TE</td>
<td>(n+1)TE</td>
</tr>
<tr>
<td>Verifier (V)</td>
<td>8TE</td>
<td>2TE</td>
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</tbody>
</table>

Table 3 1-out-of-n Transmission Loading

<table>
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<tbody>
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<td>S → R</td>
<td>n</td>
<td>p</td>
<td>+ 3n</td>
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Table 4 Comparison of Properties

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<tr>
<td>Multiple choices</td>
<td>(*1)</td>
<td>(*1)</td>
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<td></td>
</tr>
<tr>
<td>Non-reduplication</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As seen in Table 2, the proposed scheme provides the best computation loading of all schemes for all roles (signer, recipient and verifier). Table 3 shows a comparison of the 1-out-of-n transmission loading. Because this paper is designed for t-out-of-n functionality, the proposed protocol is less than ideal. Unlike conventional blind signature schemes, our scheme allows for the selection of multiple choices and provides the properties of non-reduplication, along with lower computation and communication costs (see Table 4). (*1) denotes that the multiple choices provided by Chen [12] or Tso [13] is weak and indirect that it has to proceed the signing phase process twice or more times with no guarantee of twice selection to the same item.

4. DESIGN OF SECURE MULTIPLE-CHOICE E-VOTING SCHEME

In this section, we apply our scheme to a secure mobile e-voting system. The security requirements of the e-voting system are first defined. The system framework and protocol are described. Finally we analyze our protocol according to the defined requirements.

4.1 Attacker model

In our scheme, we assume the channels between the creator and the voter, the voter and the voting center, and the creator and the voting center are insecure. Any identity (i.e. the creator, the voter, or the voting center) communicates with another via an insecure public channel, offering adversaries opportunities to intercept. In the following, we present the assumptions of the attacker model [19, 20].

(1) An adversary may eavesdrop on all communications between protocol actors over

xxxx
4.2 Security Requirements

The security requirements for the mobile e-voting system are defined as follows.

**Definition 6 (Security requirements of the mobile e-voting system)** The proposed mobile e-voting system is secure if it achieves (1) Eligibility, (2) Non-reusability, (3) Soundness, (4) Completeness, (5) Verifiability, (6) Fairness, (7) Anonymity, and (8) Non-reduplication.

The security requirements of the mobile e-voting system are as follows:

1. **Eligibility.** Only eligible voters can cast the votes.
2. **Non-reusability.** A legitimate voter can vote only once.
3. **Soundness.** No person can change other persons’ vote stealthily.
4. **Completeness.** All voters can confirm whether their votes are included in total counts.
5. **Verifiability.** No one can defraud the voting result.
6. **Fairness.** No one can get any information about the tally result before tally phase.
7. **Anonymity.** No one can determine any relationship between a vote and a voter.
8. **Non-reduplication.** No voter can select the same candidate twice.

![Fig. 4. Voting system](image-url)
4.3 Introduction of the Proposed Voting System

The scheme (as shown is Fig. 4) involves five entities: Registration center $RC$, Creator $C$, Voter $V$, Voting center $VC$, and bulletin board $BB$, where $RC$ is a trusted party. We assume that the database of Creator exist an ID list of valid voter and there exist a bulletin board which can announce vote information securely. The voting scheme includes five phases: (1) initial phase, (2) registration phase, (3) voting phase, (4) ballot-casting phase, and (5) tally phase.

The detailed steps of the protocol are as follows.

1. Initial phase

In this phase, Registration center (RC) decides a hash function $H(.)$ and each entities (including RC, C, VC) generates their public keys $(e_x, N_x)$ and private keys $(d_x, N_x)$ as follows, where $x = RC, C, VC$.

- **Step 1:** Choose two large prime numbers $p_x, q_x$.
- **Step 2:** Compute $n_x = p_x \times q_x$.
- **Step 3:** Calculate $\phi(N_x) = (p_x - 1)(q_x - 1)$.
- **Step 4:** Choose $e_x \in Z_{\phi(N_x)}^{*}$ such that $\text{GCD}(e_x, \phi(N_x)) = 1$.
- **Step 5:** Compute $d_x \in Z_{N_x}^{*}$ such that $e_x d_x = 1 \mod \phi(N_x)$.

2. Registration phase

This phase describes only eligible Voter $V_u$ can obtain valid ticket $Ticket(V_u)$ after qualification checking from the Registration center (RC). The detailed steps of the phase (shown in Fig. 5) are as follows.

- **Step 1:** $V_u$ chooses $p_{n_u} \in Z_{n_u}$ and sends $(id_u, p_{n_u})$ to RC, where $id_u$ is an identification string of $V_u$ and $p_{n_u}$ is $V_u$’s pseudo-name.
- **Step 2:** After obtaining $(id_u, p_{n_u})$, RC verifies $V_u$’s identity and voting qualification. If $V_u$ is legitimate, RC computes $s_{n_u} = H(p_{n_u})^{d_{RC}} \mod N_{RC}$ and replies $Ticket(V_u) = (s_{n_u}, p_{n_u})$ to $V_u$.
- **Step 3:** $V_u$ verify $Ticket(V_u)$ by checking whether the equation $s_{n_u}^{e_{RC}} = H(p_{n_u}) \mod N_{RC}$ is hold.

Note that $V_u$ only needs to register once for different voting issues.

![Fig. 5. Registration phase](image-url)
(3) Voting phase

This phase describes the procedures that Voter $V_u$ obtains the signatures $\sigma(m_a)$ of his/her chosen votes $m_a, j = 1, 2, \ldots, t$. The detailed steps of the phase (shown in Fig. 6) are as follows.

Step 1: $V_u$ sends $ET_a = (Ticket(\mathcal{Q}_u) \| tm_u)^c \mod N_c$ to $C$, where $tm_u$ presents current time.

Step 2: $C$ verifies whether $tm_u$ and $Ticket(\mathcal{Q}_u)$ are valid and $Ticket(\mathcal{Q}_u)$ is not reused. If it is a valid one without reusing, $C$ stores $Ticket(\mathcal{Q}_u)$ in database, chooses $r \in Z_N^*$, $C$ computes $s_i = H(m_i, r_i)^c \mod N_c$, $K_u = h(pu_u)$, $t_i = E_{K_u}(s_i \| r_i)$, and sends $\{(t_i, m_i)\}$ to $V_u$, $i = 1, 2, \ldots, n$.

Step 3: $V_u$ computes $(s_i \| r_i) = D_{K_u}(t_i)$ and verifies $H(m_i, r_i)^c \mod N_c, i = 1, 2, \ldots, n$. If all of them are correct, $V_u$ votes $m_a$, chooses $b_j \in Z_N^*$,
computes \( c_j = b_j^r \) \( \mod\ N_c \), and sends \( \{c_j\} \) to \( C \), \( i = 1, 2, \ldots, n \); \( j = 1, 2, \ldots, t \).

Step 4: \( C \) computes \( \beta_j = c_j^d \) \( \mod\ N_c \) and sends \( \{\beta_j\} \) to \( V_u \), \( j = 1, 2, \ldots, t \).

Step 5: \( V_u \) computes \( \nu_j = b_j^{-1} \beta_j \) \( \mod\ N_c \) and gets signatures \( \sigma(m_{a_j}, \nu_j) \), \( j = 1, 2, \ldots, t \).

(4) \textit{Ballot-casting phase}

This phase describes \( V_u \) sends his/her votes with signature secretly to the voting center \( VC \). The detailed steps of the phase (shown in Fig. 7) are as follows.

Step 1: \( V_u \) computes \( B_u^* = (\sigma(m_{a_j}) \| m_{a_j})^e \mod N_c \) and \( EV_u = (Ticket(\mathcal{F}_u) \| B_u^*)^e \mod N_{vc} \), and sends \( EV_u \) to \( VC \).

Step 2: \( VC \) computes \( \left( Ticket(\mathcal{F}_u) \| B_u^* \right) = D_{vc}(EV_u) \), verifies \( Ticket(\mathcal{F}_u) \), stores \( \left( Ticket(\mathcal{F}_u), B_u^* \right) \) in database, and publishes \( B_u^* \) on Bulletin board.

Step 3: \( V_u \) can check whether his/her votes \( B_u^* \) are published on Bulletin board.

(5) \textit{Tally phase}

This phase describes the tally procedures in \( VC \) when starting to tally. The detailed steps of the phase (shown in Fig. 8) are as follows.

Step 1: \( C \) publishes the private key \( d_c \) and sends it \( VC \).

Step 2: \( VC \) computes \( \left( \sigma(m_{a_j}) \| m_{a_j} \right) = (B_u^*)^e \mod N_c \), verifies whether the equation \( \left( s_u \right)^e = H(m_{a_j}, (\nu_j)^e) \mod N_c \) hold, and publishes \( d_c \), the tally result and \( B_u^* \) if which is valid.

Step 3: Each person can verify the validity of \( B_u^* \).

\begin{align*}
\text{Voter} & \quad \text{Voting center} \\
B_u^* & = (\sigma(m_{a_j}) \| m_{a_j})^e \mod N_c \\
EV_u & = (Ticket(\mathcal{F}_u) \| B_u^*)^e \mod N_{vc} \text{ compute } (Ticket(\mathcal{F}_u) \| B_u^*) = D_{vc}(EV_u) \\
& \text{verify } Ticket(\mathcal{F}_u) \\
& \text{publish } B_u^*
\end{align*}

Fig. 7. Ballot-casting phase

\begin{align*}
\text{Creator} & \quad \text{Voting center} \\
& \quad d_c \\
& \text{compute } (\sigma(m_{a_j}) \| m_{a_j}) = (B_u^*)^e \mod N_c \\
& \text{verify } (s_u)^e = H(m_{a_j}, (\nu_j)^e)
\end{align*}

Fig. 8. Tally phase

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4.4 Security Analysis of the Proposed E-Voting

We analyze our protocols according to the requirements defined in Section 4.2.

1. Eligibility. In the registration phase, RC verifies $\mathcal{V}_u$’s voting qualification by his/her identity, only legitimate voters $\mathcal{V}_u$ can get $Ticket(\mathcal{V}_u)$ from RC, and $Ticket(\mathcal{V}_u)$ is required in the voting phase.

2. Non-reusability. In Step 2 of the voting phase, C checks if $Ticket(\mathcal{V}_u)$ is stored in the database, if it is, C aborts the process, preventing $\mathcal{V}_u$ from reusing $Ticket(\mathcal{V}_u)$.

3. Soundness. In the tally phase, all valid ballots are published on the Bulletin board and can be verified publicly with the public key $e$, which prevents invalid ballots.

4. Completeness. In the ballot-casting phase, VC verifies $Ticket(\mathcal{V}_u)$ and publishes $B_j^v$ on Bulletin board if $Ticket(\mathcal{V}_u)$ is legitimate. All $\mathcal{V}_u$s can check whether his/her ballots are included on Bulletin board to prevent incompleteness.

5. Verifiability. The entire final ballots are posted publicly and can be verified with the public key $e$.

6. Fairness. There is not any person who can obtain any knowledge about the tally before the tally phase. Attackers, including the signer C, cannot know any information about the chosen votes in the voting phase due to the privacy of oblivious signature, of which the security analysis is provided in section 3.4. Moreover, attackers, cannot know any information about the ballot in the ballot-casting phase due to the encryption of messages, of which the security can be proved via Def. 7 and Thm. 5

Definition 7 (partial RSA problem) Let $(e, N)$ be RSA public keys and $c = m' \mod N$, where $m, c \in \mathbb{Z}_n$ and $m = m_l \| m_r$. If $m_s$ can be evaluated from given $e, N$ and $c$, then we say partial RSA problem can be solved. (The probability of solving this problem is denoted as $Pr(m_l \mid e, N, c = \varepsilon_{rsa})$.)

Theorem 5 (Fairness) In our protocol, if a ballot can be obtained in the ballot-casting phase, then the partial RSA problem can be solved.

Proof. An adversary tries to obtain a ballot by evaluating $m_{s_j}$ from given $e_c$, $B_j^v$ and $N_c$. Let $RO$ be a random oracle: input $e_c$, $B_j^v$ and $N_c$ to output $m_{s_j}$ such that $B_j^v = (\sigma(m_{s_j}) \| m_{s_j})^{\gamma} \mod N_c$ (i.e. $RO(e_c, B_j^v, N_c) \Rightarrow m_{s_j} : B_j^v = (\sigma(m_{s_j})^{\gamma} \mod N_c)$. In definition 7, Let $e_c \leftarrow e$, $B_j^v \leftarrow c$ and $N_c \leftarrow N$ be input parameters of $RO$ and obtain output $m_{s_j}$. Let $m_z \leftarrow m_{s_j}$, then $m_z$ is evaluated. Therefore, $Pr(m_l \mid e_c, N_c, B_j^v) \leq Pr(m_l \mid e, N, c = \varepsilon_{rsa})$, which means the partial RSA problem can be solved if $RO$ exists.

7. Anonymity. No one can determine any relationship between a vote and a voter since each voter uses his/her own one-time pseudo name in the whole process.

8. Non-reduplication. If $\mathcal{V}_u$ attempts to vote the same candidate twice, he/she can
randomly choose \( b_1, b_2 \) in the voting phase and calculate
\[
c_1 = b_1' r_i \mod N,
\]
\[
c_2 = b_2' r_i \mod N.
\]
However, after extracting the signature parameters \( v_1, v_2 \), it turns out
\[
v_1 = b_1^{-1} \beta_i = v_2 = b_2^{-1} \beta_i = r_i' \mod N,
\]
the voter still gets the same signature on the same candidate, preventing any voter from selecting the same candidate twice.

### 4.5 Comparison

There are lots of works for e-voting [21-26]. In the section, we compare some properties of the voting schemes in different methods including (A) Traditional voting, (B) direct authorization [21,22], (C) anonymous identifiers [21], (D) blind signatures [21,23], (E) oblivious signatures [12,13], and (Ours) \( t \)-out-of-\( n \) oblivious signatures.

As seen in Table 5, the proposed scheme provides the functions for all properties. Unlike conventional blind signature schemes and oblivious signatures, our scheme allows for multiple selections of multiple choices and provides the properties of non-reduplication. Similar with Table 4, \((*)1\) denotes that the property of multiple choices provided by oblivious signatures is weak and indirect that it has to proceed the signing phase process twice or more times with no guarantee of twice selection to the same item.

### 5. IMPLEMENTATION ON MOBILE SYSTEM

We implement a simulation prototype based on the proposed mobile e-voting scheme on mobile phones running the Android operating system. We use the SHA-256 and the RSA public key system to implement the hash function and encryption/decryption algorithms. The transmission interface is Wi-Fi.

<table>
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<th>Protocols</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
<th>(Ours)</th>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>( t ) choices out of ( n ) candidates</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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</tr>
<tr>
<td>Non-reduplication</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

(A) traditional voting,
(B) direct authorization,
(C) anonymous identifiers,
(D) blind signatures,
(E) oblivious signatures,
(Ours) our proposed scheme.
Fig. 10. Application flowchart
To implement our proposed scheme, we used a Wi-Fi AP and five cell phones (shown in Fig. 9). Fig. 10 shows the scheme flow in terms of data transmission and Fig.
Fig. 11 shows the Android mobile phone screens of the JAVA prototype implementation. The hardware included two HTC models (Desire S and Desire HD) both running the Android 2.3.5 Professional OS with 1GHz CPU. The implementation included three roles: Creator, Voter, and Voting center.

Fig. 11 (a) illustrates "Role choice" when the application starts running. There are three roles, Creator, Voter, and Voting center, can be chosen. If we click "Voter" or "Voting center" (shown in Fig. 11 (b) and Fig. 11 (c)), it presents a listening (or waiting) state to wait the messages sent from Creator. If we want to create a voting, we can choose "Creator", and input Vote issue, single/multiple choice, candidates, etc. Fig. 11 (d) illustrates multiple choice (three choices).

After pressing "Send" and "Yes" (shown in Fig. 11 (e)), Creator sends voting messages to Voter and goes to listening (or waiting) state. After obtaining messages (shown in Fig. 11 (f)), Voter presses "Verify" to verify the validity of the messages, chooses the candidates he/she want to vote (shown in Fig. 11 (g)), and presses "Send back to creator" to send the messages of chosen candidates to Creator for processing oblivious signatures.

After getting messages and processing oblivious signature operations, Creator sends the corresponding messages back to Voter. Next, after Voter gets messages, the chosen candidates are illustrated in red and the button "Send to voting center" appears (shown in Fig. 11 (h)).

Next, Voter press the button "Send to voting center" to wait vote result (under listening mode). When voting time is up (shown in Fig. 11 (i)), Creator can press "Yes" button. After getting the message from Creator (shown in Fig. 11 (j)), Voting center press "Verify" button to verify if the tally is valid. Next, Voting center keeps valid tallies and drops invalid ones, and the button "Tally" appears (shown in Fig. 11 (k)). Finally, voting center press "Tally" button. The poll bar chart of final result appears (shown in Fig. 11 (l)) and this vote finishes.

6. CONCLUSION

This paper constructs a t-out-of-n oblivious signature scheme that satisfies the security properties of completeness, unforgeability, privacy, selection restriction and non-reduplication. Security analysis and comparisons of computation and communication performance validate the capability and efficiency of the proposed protocol, and its suitability for use in anonymous electronic voting applications. A mobile e-voting protocol using the proposed t-out-of-n oblivious signature scheme are also proposed and the implementation of the proposed e-voting protocol on Android system mobile devices allows users to securely use the mobile e-voting system conveniently. Future work will focus on decreasing the computation cost for recipients.

REFERENCES


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