The spectrum allocated to an end-to-end connection between the source and destination nodes varies dynamically with time (denoted as time-varying traffic). For serving time-varying traffic in an elastic optical network (EON), the spectrum allocated for the connection can be expanded or contracted to meet the traffic requirement. Moreover, using multipath routing scheme can more flexibly utilize spectrum resources than using single-path routing in a dynamic scenario, since it splits a traffic request into multiple small size connections and individually transmitting them through several optical paths. In this paper, the Spectrum Expansion/Contraction Problem (SECP) for multipath routing on EONs with time-varying traffic is studied. The path selecting policy (PSP), expansion and contraction algorithms are proposed and examined through simulations. Simulations show that the proposed algorithms can achieve good results.

Keywords: spectrum expansion/contraction, elastic optical network, multi-path routing, time-varying traffic

1. INTRODUCTION

Elastic optical networks (EONs), which employ optical-orthogonal frequency division multiplexing (O-OFDM) technology, have been proposed to scale the demands by efficiently utilizing the spectrum as they provide finer spectrum granularity and distance adaptive modulation formatting. The spectrum of a link in EONs is divided into small unit sub-carriers or frequency slots (FSs) and the necessary amount of consecutive FSs are assigned to support the connection request. Besides, more efficient spectrum allocation is achieved in these networks due to flexible grid and elastic line rates providing finer granularity [1, 2]. EONs also provide a super-channel connectivity for accommodating ultra-high capacity demands and a sub-wavelength granularity for low-rate transmissions [1, 2]. Hence, O-OFDM can achieve subwavelength granularity, by using elastic bandwidth (BW) allocation that manipulates the FSs. Specifically, a BW-variable O-OFDM transponder can assign an appropriate number of FSs to serve a lightpath request using just-enough FSs. On EONs, we need to develop routing, modulation-level, and spectrum assignment (RMSA) algorithms for network control and management [3]. For a given connection request, the goal of the RMSA is to find a lightpath on the network and assign the FSs, it has emerged as the essential problem for spectrum management in EONs.
Fig. 1. (a) Example of multipath routing, (b) spectrum allocation for multi-path, (c) frequency slices and allocation on EONs, (d) elastic scheme with spectrum expansion/contraction.

1.1 Multipath Routing

For online provisioning, it is difficult to serve certain large BW requests with single-path routing due to the BW limitation, thus resulting in high request blocking probability [4]. The elastic nature of EON enables us to split a connection’s traffic over multiple routing paths without causing significant bandwidth waste. In [4], authors proposed two dynamic service provisioning algorithms that incorporate a hybrid single-/multi-path routing (HSMR) scheme. The simulation results have demonstrated that the proposed RMSA-HSMR algorithm can effectively reduce the blocking probability of dynamic RMSA, as compared to algorithms that use single-path routing. In Figure 1(a), the connection request $R = (A, D, 4)$, between nodes $A$ and $D$ required four FSs, can be achieved by two routing paths $p_1$ and $p_2$. The allocated FSs for these lightpaths are shown in Figure 1(b). If multipath routing were not allowed, the request is blocked.

In [5], authors proposed a novel dynamic multi-path service provisioning algorithm that is specifically designed for EONs and considers the differential delay constraint. In [6–9] authors investigated the survivable routing problem. In [6], authors used multipath routing and the p-cycles protecting scheme to route the connection request. In [7], authors proposed a new recovery scheme, called multipath recovery (MPR). In [8], authors proposed multipath recovery and bitrate squeezing technique to maximize the amount of restored bitrate. In [9], authors used multi-path routing scheme, one path is the primary path and the other are the backup paths [9]. In [10], authors investigated the potential gains by jointly employing the multipath routing and traffic grooming techniques in combination with a realistic physical impairment model.

1.2 Time-Varying Traffic

In a network, each optical connection can be transmitted by an allocated channel which consists a central frequency (CF) and a size. The size of the channel is determined by the requested bit-rate, the modulation technique applied, the (fixed) slice width and the guard band (GB) introduced to separate two spectrum adjacent connections, among others. In Figure 1(c), the CF of the lightpath is set to 193.1 THz and size is set to 6 FSs with BW 37.5 GHz (1 FS = 6.25 GHz).

Several spectrum allocation schemes that change the bandwidth dynamically have been studied in [11–15]. In [11] authors used two connections adjacent to share the
optical spectra. A general policy to allocate FSs to time-varying traffic was presented in [12–15]. In [13], authors defined a general spectrum allocation framework for time-varying traffic demands on EONs. Namely, they discerned three SA schemes of different levels of elasticity. (1) **Fixed**: both the assigned CF and the size do not change in time. (2) **Semi-Elastic**: the assigned CF is fixed, but the size may vary. (3) **Elastic**: both the assigned CF and the size can be subject to change by performing Spectrum Expansion/Contraction (SEC) in each time interval (in Figure 1(d)). In this paper, the elastic scheme with expansion/contraction is used for the multi-path routing on EONs with time-varying traffic.

In [16, 17] authors have studied different spectrum reallocation techniques: **make-before-break** and **push-pull** on EONs with time-varying traffic. In [18], two SEC schemes, constant spectrum allocation (CSA) and dynamic high expansion-low contraction (DHL), were developed. In [19], five SEC schemes are proposed; they are CSA, dynamic alternate direction (DAD), avoid close neighbors (ACN), shift-ACN and float-ACN. In [20], authors extended **defragmentation** algorithm on EONs with time-varying traffic. Furthermore, the EONs enable to expand/contract slot width of channel [21]. Future EONs will change slot width according to time-varying traffic by changing the number of FSs flexibility. In [21], authors proposed a **grouped-elastic spectrum allocation** scheme to satisfy the tolerable waiting time of the service in an optical fiber link.

In this paper, the **Spectrum Expansion/Contraction Problem (SECP)** for multipath routing on EONs with time-varying traffic is studied. To the best of our knowledge, this is the first paper that focuses on the SECP for multipath routing of unicast demands in EONs. The rest of the paper is organized as follows. First, in Section 2 the definition and assumptions of the problem are given, and the RMSA-HSMR algorithm for multipath computation proposed in [4] is stated. In Section 3 the proposed algorithms are described (includes SEC algorithm and the path selecting policy (PSP). Then, in Section 4, the performance of the proposed methods is examined. Finally, the conclusion is drawn in Section 5.

### 2. STUDIED PROBLEM

In this paper, the SECP for multipath routing on EONs with time-varying traffic is studied. When the request arrived, if it is a new connection request, then the RMSA-HSMR algorithm [4] is performed to find the set of routing paths (single/multipath) and the assigned FSs. If the required bandwidth cannot be satisfied, then the request is blocked; Otherwise, the lightpaths are established and the required FSs of lightpaths are allocated. If the request is an adjusted request, based on the selected spectrum expansion/contraction policy, the CF and the size of the lightpaths are adjusted (expanded or contracted). If the rate variation can be accommodated, then the connections and rates of the lightpaths are adjusted and accepted. Otherwise, the RMSA-HSMR algorithm is performed to route the excess traffic or reroute all lightpaths of the connection. The studied problem is for the hybrid single/multiple paths routing scheme, if the traffic of the request varies, how to adjust the lightpaths so that the network performance can be optimized.

Consider the example shown in Figure 1(a), if the connection \( R_7(A,D,1\text{ FSs}) \) arrived, it means that the bandwidth of the request \( R_7 \) decreases from 4 FSs to 1 FSs. Since two lightpaths \( p_1 \) and \( p_2 \) are used to route the request. One of the lightpaths (\( p_1 \) or \( p_2 \)) should be deleted and the other lightpath should be contracted. For a given EON and a sequence of requests, the goal is to add/delete/expand/contract lightpaths and assigned suitable channels to the lightpaths to meet the traffic requirement such that the perfor-
mance measure can be optimized. In this paper, two algorithms are proposed to solve this problem. In the following, the assumptions, constraints, notations and the definitions of the studied problem are given.

2.1 Assumptions

The assumptions of the studied problem are given as follows. For each link, there is a fiber connecting the end-nodes and signal can be transmitted bidirectionally. All nodes in the network are equipped with bandwidth variable wavelength cross-connects (BV-WXC). For simplicity, the numbers of FSs provided by links are all equal. The bandwidth requirement between nodes can be transmitted by using multiple lightpaths with same or different routes and numbers of FSs.

2.2 Constraints

In this section, for a given lightpath, the same block of FSs of every link along the lightpath is allocated (called as spectrum continuity constraint). Sub-carriers of the same data stream must be consecutive along the frequency domain (called as sub-carrier consecutiveness constraint) [1, 2]. Allocated frequency slots for paths must be separated by guard band (GB) in order to prevent interfering, i.e., at least one FS must be assigned as a GB between the FS set of every lightpaths. Likewise, this constraint also implies that one frequency slot can be employed by only a single lightpath at a time (called as non-overlapping spectrum assignment constraint) [1, 2].

To avoid a request from being split over too many paths, a BW allocation granularity (denoted as g slots) is defined [4]. Specifically, when the request is provisioned over more than one routing paths, the minimum number of FSs which can be allocated on each path is g. Note that increasing g eventually lead to a single-path-only scenario when g is comparable to the largest size of the requests [4].

2.3 Notations

Let \( G = (V, E, B, d) \) be the physical topology of the network, where \( V = \{v_1, v_2, ..., v_n\} \) is the set of nodes (|V| = n), \( E = \{e_1, e_2, ..., e_m\} \) is the set of links, \( B \) is the number of FSs provided by each fiber, and \( d(e_i) \) is length of the link \( e_i \in E \). We assume that the BW of each FS is unique as \( BW_{sl} \) GHz [4]. Let \( M \) be the modulation level in terms of bits per symbol; \( M \) can be 1, 2, 3, and 4 for BPSK, QPSK, 8-QAM, and 16-QAM, respectively. \( C_{sl} \) represents the capacity of a slot when the modulation is BPSK \((M=1)\) and \( BW_{sl} \) is equal to \( M \times C_{sl} \). \( R(s, d, C) \) is the request, where \( s \in V \) and \( d \in V \) is the source and destination node of the lightpath, respectively; and \( C \) is the required bandwidth (the unit of \( C \) is GHz). For each link \( e_i \) in \( E \), a B-bit bit-mask \( b_i(j), j = 1, 2, ..., B \), represents the status of the \( j^{th} \) FS of the link \( e_i \), \( e_i \in E \); if the \( j^{th} \) FS is occupied, then \( b_i(j) = 1 \); otherwise, \( b_i(j) = 0 \). Let \( \sum(e_i) \) be the summation of the bit-mask of the link \( e_i \) for all \( j = 1, 2, ..., B \), that is, \( \sum(e_i) = \sum_{j=1}^{B} b_i(j) \).

For a lightpath request \( R(s, d, C) \), the RMSA-HSMR algorithm proposed in [4] with online path computation is used to determine a set of routing paths \( P = \{p_{s,d,i}|i = 1,2, ..., z\} \) to serve the request, where \( i \) is the index of each routing path. Note that, for different \( i \), the routing paths can be identical but their spectrum allocations are not contiguous. For each path \( p_{s,d,i} \in P \), the length of path \( (lop) \) can be computed by \( lop(p_{s,d,i}) = \sum_{e_i} d(e_i) \), \( e_i \in p_{s,d,i} \). The modulation level of the path can be derived as \( M_i = ML(lop(p_{s,d,i})) \), where \( ML() \) returns the highest modulation level that a transmission distance can support. Specifically, we assume that each modulation \( M \) can support a
maximum transmission distance based on the receiver sensitivities [4], and when the distance of \( p_{s,d,i} \) permits, we always assign the highest modulation level to guarantee high spectral efficiency [4].

### 2.4 RMSA-HSMR Algorithm

In this subsection, the details of the RMSA-HSMR algorithm proposed in [4] are described. First, for a new connection request \( R(s,d,C) \), the status of the links of the current network are updated. The network \( G(V,E,B,d) \) is then transformed to a graph \( G'(V,E,d') \), where the weight \( d'(e_l) \) of link \( e_l \) is calculated according to the link spectrum usage and by using the formula

\[
d'(e_l) = \begin{cases} +\infty, & \text{MaxBlock}(e_l) \leq g \\ w_l \times \frac{\text{sum}(b_l)+g}{B}, & \text{MaxBlock}(e_l) > g, \end{cases}
\]

where MaxBlock(\( e_l \)) is the maximal number of continuous free FSs on link \( e_l \), and \( w_l \) is calculated from \( d(e_l) \) with \( w_l = ML_{max} - ML(d(e_l)) + 1 \), where \( ML_{max} \) is the highest modulation level that can be supported in the network [4]. A link \( e_l \) is omitted from the online path computation if it does not have a block of available contiguous slots with the size \( > g \). Otherwise, the link weight \( d'(e_l) \) is proportional to the product of \( w_l \) and the number of used slots \( \text{sum}(b_l) \) [4].

For the connection request \( R(s,d,C) \), the \( K \)-shortest paths algorithm [22] is performed on the network \( G'(V,E,d') \) to find a set of candidate paths \( CP = \{(p_{s,d,i}), i = 1,2,...,K \} \). Paths in \( CP \) are sorted in increasing order according to the length of the paths \( lop(p_{s,d,i}) = \sum_{e_l \in p_{s,d,i}} d(e_l) \). After the candidate path is selected, the last step of RMSA-HSMR is the spectrum assignment to finalize the allocations of contiguous FSs along the fiber links on \( p_{s,d,i} \). The spectrum assignment on \( p_{s,d,i} \) is to find \( N_i \) contiguous bits based on current value \( b_l \) of link \( e_l \) in \( p_{s,d,i} \). The set of routing paths for request \( R(s,d,C) \) is denoted as \( P = \{(p_{s,d,i},M_i,N_i,C_i), i = 1,2,...,z \} \). The bandwidth \( C_i \) allocated to the selected path \( p_{s,d,i} \in P \) is determined based on the network status and should satisfy the constraint \( C \leq \sum_{p_{s,d,i} \in P} C_i \). The number of assigned contiguous FSs \( N_i \) for the path \( p_{s,d,i} \) is computed by \( N_i = \lceil C_i/(M_i \times C_{gb}) \rceil + N_{GB} \), where \( N_{GB} \) is the number of FSs for the GB. Note that when the traffic is routed by multiple paths, more FSs will be used for the GB. In the context of this study, we assume that \( N_{GB} = 1 \) and this guard band is inserted as the highest indexed slot in the spectrum assignment of each connection. The request \( R \) is blocked, if a feasible set of routing paths \( P = \{(p_{s,d,i},M_i,N_i,C_i), i = 1,2,...,z \} \) cannot be found. The detailed of the RMSA-HSMR algorithm are shown in Algorithm 1.

### 3. SEC ALGORITHM

In this section, the Path Selecting Policy (PSP) and two algorithms (named as Expansion Algorithm and Contraction Algorithm) are proposed to do the SEC operation. The proposed algorithms are described in the following subsections.

#### 3.1 Path Selecting Policy (PSP)

When an adjusted connection request, since the request may be currently routed by multipath, several lightpaths can be selected to expand or contract to meet the traffic requirement. However, the path selecting policy should be designed to select the current lightpaths for expansion or contraction. The priority or sequence of the selected paths can be determined by the PSP.
Algorithm 1 RMSA-HSMR Algorithm [4]

1: Input: $G(V, E, B, d), R(s, d, C)$
2: Output: the set of lightpaths $P = \{(p_{s,d,i}, M_i, N_i, C_i), i = 1, 2, ..., z\}$
3: $P = \emptyset$, collect link status of network $G(V, E, B, d)$ and restore network resources used by expired requests
4: update link weights $d'(e_i), \forall e_i \in E$ based on the current network status, using formula (1);
5: construct graph $G'(V, E, d')$ and read the incoming request $R(s, d, C)$;
6: calculate a set $CP = \{p_{s,d,i}, i = 1, 2, ..., K\}$ of $K$-shortest routing paths from $s$ to $d$ on $G'(V, E, d')$;
7: sort the paths in $P$ in ascending order based on the distances $lop(p_{s,d,i}) = \sum_{\forall e_i \in p_{s,d,i}} d'(e_i)$.
8: for all $(p_{s,d,i} \in CP)$ do
9:   determine the highest modulation level $M_i$ for the path $p_{s,d,i}$ with its real distance $lop(p_{s,d,i})$;
10: for all (all available slot blocks with sizes $> g$) do
11:   allocate maximal allocatable capacity $C_i$ to slot blocks with $N_i$ and add path $(p_{s,d,i}, M_i, N_i, C_i)$ to $P$;
12: if $\sum_{\forall e_i \in p_{s,d,i}} C_i \geq C$ then
13:   break inner and outer for-loops;
14: end if
15: end for
16: end for
17: if $\sum_{\forall e_i \in p_{s,d,i}} C_i < C$ then
18:   revert all the spectrum allocations and remove all paths from $P$, mark the request as block;
19: return false;
20: end if
21: return success and $P = \{(p_{s,d,i}, M_i, N_i, C_i)\}$

The PSP for Expansion Algorithm are listed as follows:

- **Maximal Expansible Slots First** (MaxESF): The path with a maximal number of expansible slots is selected first. For each lightpath, there are two possible expansible directions: higher index and lower index directions. For each link $e_i \in p_{s,d,i}$, the maximal expandable number of free FSs for higher and lower index direction is denoted as $HI(e_i)$ and $LW(e_i)$, respectively. The maximal number of expansible slots on a path can be computed by $MaxES(p_{s,d,i}) = \min_{\forall e_i \in p_{s,d,i}} \{|HI(e_i)| + \min_{\forall e_i \in p_{s,d,i}} \{|LW(e_i)|\}\}$. Select the path with the maximal expansible slots can satisfy the bandwidth requirement quickly and reduce the number of required GBs.

- **Shortest Hop First** (SHF): We select the paths in the ascending order based on the number of hops $hop(p_{s,d,i})$ of the path. Select the path with the minimal hop can be expanded to satisfy the bandwidth requirement as small as possible.

- **Minimal Fragmentation Ratio First** (MinFRF): The routing path with minimal fragmentation ratio will be selected first. The link fragmentation ratio is usually introduced to describe the spectrum occupation of link [1]. The fragmentation ratio of a path $p_{s,d,i}$ is the ratio of the summation of the maximal number of continuous free frequency-slots $MaxBlock(e_i)$ on link $e_i \in p_{s,d,i}$ to the summation of the number of free frequency-slots on link $e_i \in p_{s,d,i}$. It can be computed by the following formula: $FR_{p_{s,d,i}} = \sum_{\forall e_i \in p_{s,d,i}} MaxBlock(e_i) / \sum_{\forall e_i \in p_{s,d,i}} (B - \text{sum}(e_i))$.

The PSPs for Contraction Algorithm are listed as follows:

- **Maximal Fragmentation Ratio First** (MaxFRF): The routing path with maximal fragmentation ratio will be selected first. This is the opposite selection for the expansion.

- **Minimal Weighted Hop First** (MinWHF): We select the paths in the ascending order based on the weighted hop of the path $p_{s,d,i}$: $WH = (|V| - hop(p_{s,d,i})) \times N_i \times M_i$. In the WH measure, the modulation level, the number of allocated FSs, and the number of hops of a path are taken into consideration. The smaller one is selected first to reduce the bandwidth.
Algorithm 2 Expansion Algorithm

1: Input: $G(V, E, B, d), P = \{(p_{s,d,i}, M_i, N_i, C_i) | i = 1, 2, \ldots, z\}$, $R(s, d, C_{new})$, path selecting policy (PSP);
2: Output: $P_{new}$.
3: Restore network resources used by expired requests and let $P_{new} = \emptyset$.
4: Calculate the total bandwidth provided by the set of current lightpaths $C' = \sum_{(p_{s,d,i}) \in P} C_i$ and difference $C'' = C_{new} - C'$.
5: while ($C'' > 0$) do
6:  
7:  Select and remove a path $p_{s,d,i} \in P$ with highest priority according to the PSP.
8:  if (path $p_{s,d,i}$ can be found) then
9:     // expand path $p_{s,d,i}$
10:     Find the maximal expandable FSs $MaxES(p_{s,d,i})$ for the selecting lightpath $p_{s,d,i}$.
11:     Compute the number of required FSs $FS_e = \lceil ML(\log(p_{s,d,i})) \times C_i \rceil$.
12:     Determine the number of FSs to be expanded $F_p = \min\{MaxES(p_{s,d,i}), FS_e\}$.
13:     Determine the bandwidth $C_{p} = C_{p}' + ML(\log(p_{s,d,i})) \times C_i$.
14:     Update $C'' = C'' - C_{p}' = C'' - F_p \times ML(\log(p_{s,d,i})) \times C_i$.
15:     Expand the channel size and adjust CF of the lightpath $p_{s,d,i}$, then add $p_{s,d,i}$ to $P_{new}$.
16:     Restore paths in the original set $P$, block the request and return success and $P_{new}$.
17:  
18:  else
19:     // $P$ is empty and cannot be expanded, add new lightpaths
20:     Perform RMSA-HSMR algorithm to find a set $P'$ of new lightpaths with bandwidth $C''$ between nodes $s$ and $d$.
21:     Perform RMSA-HSMR algorithm to find a set $P_{new}$ of new lightpaths with bandwidth $C_{new}$ between nodes $s$ and $d$.
22:     if (the set $P'$ can be found) then
23:      $P_{new} = P_{new} \cup P'$ and return success and $P_{new}$.
24:     else
25:      Temporarily release all resources possed by current lightpaths in set $P_{new}$.
26:      Perform RMSA-HSMR algorithm to find a set $P_{new}$ of new lightpaths with bandwidth $C_{new}$ between nodes $s$ and $d$.
27:      if (the set $P_{new}$ can be found) then
28:       return success and $P_{new}$.
29:      end if
30:      Restore paths in the original set $P$, block the request and return false.
31:     end if
32:  end if
33: end while
34: $P_{new} = P_{new} \cup P$ and return success and $P_{new}$.

3.2 Expansion Algorithm

When the bandwidth of the request increases, the bandwidth of current supporting lightpaths should be increased or a (or several) new lightpath(s) with suitable modulation level should be established with required number of FSs. Since the FSs expansion of the current path can reduce the FR ratio and get better spectrum efficiency, the current lightpaths are expanded first, if possible. After performing expansion, if the required bandwidth cannot be fully supported, a set of new lightpaths is added to route the unsupported bandwidth. This set of new paths is found by performing the RMSA-HSMR algorithm.

We assume that the new connection request is represented by $R(s, d, C_{new})$. The set of current lightpaths used to support the request is represented by $P = \{(p_{s,d,i}, M_i, N_i, C_i) | i = 1, 2, \ldots, z\}$, where $C_i$ is the bandwidth provided by the lightpath $p_{s,d,i}$, $M_i$ is the modulation level of the lightpath $p_{s,d,i}$, $N_i$ is the number of allocated FSs and $C' = \sum_{(p_{s,d,i}) \in P} C_i$ represents the total bandwidth for the current connection request. If the bandwidth of the updated request increases, i.e., $C_{new} > C'$ (or $C'' = C_{new} - C' > 0$), then the Expansion Algorithm will be performed.

Based on the selected path selecting policy, a lightpath $p_{s,d,i}$ in $P$ is selected and removed to expand (steps 7–17). If the selected path can be expanded, then find the maximal number of expandable FSs (denoted as $MaxES(p_{s,d,i})$ and defined in Section 3.1) and the required number of FSs (denoted as $FS_e$, and $FS_e = \lceil ML(\log(p_{s,d,i})) \times C_{lop} \rceil$). Then, the actual number of FSs to be expanded (denoted as $F_p$) on the selected path $p_{s,d,i}$
is determined by $F_p = \min \{MaxES(p_{s,d,i}), FS_e\}$. If $F_p$ is equal to $FS_e$, then update $C^e$ to 0 and terminate the expansion process. If $F_p$ less than $FS_e$, then update $C^e$ by subtracting $F_p \times ML(lo(p_{s,d,i})) \times C_{st}$ and repeat the expansion process.

If there is no path $p_{s,d,i} \in P$ can be expanded (steps 19–23), then the remaining required bandwidth $C^e$ will be supported by new lightpaths found by performing the RMSA-HSMR algorithm. If all required bandwidth can be supported by the set $P'$ of new paths, then return $P^{\text{new}} \cup P'$.

If the bandwidth requirement cannot be supported by performing previous steps, then all currently deployed lightpaths are temporarily released (steps 25–31). The connection request will be treated as a new request and the required bandwidth will be supported by new lightpaths found by performing the RMSA-HSMR algorithm. If the set $P^{\text{new}}$ can be found, then return success. Otherwise, block the request and restore all original paths in the original set $P$. The details of the Expansion Algorithm are described in Algorithm 2.

### 3.3 Contraction Algorithm

When the bandwidth of the request decreases, it can be achieved by performing spectrum contraction or lightpath deletion. In this paper, the allocated FSs of current lightpaths are deleted first, if possible. We assume that the new request is represented by $R(s,d,C^{\text{new}})$. The set of current lightpaths used to support the original request $R(s,d,C)$ is represented by $P = \{(p_{s,d,i}, M_i, N_i, C_i), i = 1, 2, ..., z\}$ and $C' = \sum_{p_{s,d,i} \in P} C_i$ is the total bandwidth provided by current set $P$ of lightpaths.

It is worth noting that, the total bandwidth provided by the set of lightpaths are greater or equal to the original required bandwidth $C$, i.e., $C' \geq C$. Since the minimal unit of bandwidth allocation is a single FS, and the smallest bandwidth provided by a slot is $C_{st}$ for the case with modulation level $M_i=1$. If the path $p_{s,d,i}$ with higher modulation level (with shorter distance) is contracted, the reduced bandwidth of a slot is greater than $C_{st}$. Actually, the reduced bandwidth is equal to the $M_i \times C_{st}$ for the path $p_{s,d,i}$ per slot. However, in contraction, it is important that not to over-contracting the bandwidth by selecting suitable lightpath and determining the number of required FSs. Moreover, after contraction, the number of allocated FSs of the lightpath should be greater than or equal to BW allocation granularity $g$.

If $C^{\text{new}}$ is equal to zero, all lightpaths in $P$ for the request are deleted. Otherwise, we have to make sure that the contraction needs to be performed. That is if the required bandwidth is less than or equal to the bandwidth of the total bandwidth provided by current paths subtract by $C_{st}$. It means that if $C' - C^{\text{new}} - C_{st}$ is greater than zero ($C' - C^{\text{new}} - C_{st} > 0$), then the lightpath contraction process should be performed. The maximum bandwidth will be reduced is $C = C' - C^{\text{new}}$. Based on the path selecting policy, a lightpath $p_{s,d,i}$ in $P$ is selected to contract.

For the selected path $p_{s,d,i} \in P$, there are several cases should considered in performing contraction.

- If $(C^c \geq C_i)$, then all bandwidth of the selected path $p_{s,d,i}$ is contracted, path $p_{s,d,i}$ is removed from $P$ and the values of $C^c$ and $C'$ are updated.

- If $(C_i \geq C^c \geq C_i - (g \times M_i \times C_{st}))$, after contraction, the bandwidth of path $p_{s,d,i}$ remains $g \times M_i \times C_{st}$. This case can be described by the example shown in Figure 2(a). $N_i - g$ is the number of FSs to be contracted from lightpath $p_{s,d,i}$. After contraction, we have $C^c = C' - (N_i - g) \times M_i \times C_{st}$ and $C' = C' - (N_i - g) \times M_i \times C_{st}$. Then lightpath $p_{s,d,i}$ is updated and removed from $P$, and then add to $P^{\text{new}}$. 

• If \((C_i - (g \times M_i \times C_{d_i}) > C^c \geq g \times M_i \times C_{d_i})\), then partial bandwidth of path \(p_{s,d,i}\) can be contracted to meet the requirement. This case can be described by the example shown in Figure 2(b). The bandwidth of the lightpath \(p_{s,d,i}\) is reduced to \(C' = \left\lfloor \frac{C_i - C^c}{g} \right\rfloor \times M_i \times C_{d_i}\). Then, \(C^c = C^c - C_p\) and \(C'\) are updated so that the Contraction Algorithm can terminate successfully. Final, lightpath \(p_{s,d,i}\) is updated and removed from \(P\), and added to \(P^{new}\).

The details of the Contraction Algorithm are described in Algorithm 3.

Algorithm 3 Contraction Algorithm

1: **Input:** \(G(V,E,B,d)\), \(P = \{(p_{s,d,i},M_i,C_i)\} (i = 1,2,...,z)\), \(R(s,d,C^{new})\), path selecting policy (PSP);
2: **Output:** \(P^{new}\);
3: 4: Restore network resources used by expired requests. \(P^{new} = \emptyset\).
5: if \((C^{new} = 0)\) then
6: Remove all lightpaths in \(P\), restore network resources of lightpaths and **return** \(P^{new}\).
7: end if
8: Calculate the total bandwidth provided by the set of current lightpaths \(C' = \sum_{p_{s,d,i}<P} C_i\).
9: while \((C' - C^{new} - C_d > 0)\) do
10:   \(C' = C' - C^{new}\);
11: Select a path \(p_{s,d,i}\) in \(P\) with highest priority according to the PSP.
12: if \((C' \geq C_i)\) then
13:   Remove lightpath \(p_{s,d,i}\) from \(P\), release resource of \(p_{s,d,i}\), and update \(C' = C' - C_i\) and \(C' = C' - C_i\).
14: else if \((C_i > C' > C_i - (g \times M_i \times C_{d_i}))\) then
15:   // all but bandwidth with g FSs of path \(p_{s,d,i}\) can be contracted
16:   Determine \((N_i - g)\) the number of FSs to be contracted for lightpath \(p_{s,d,i}\).
17:   Reduce the bandwidth of the lightpath \(p_{s,d,i}\), to \(g \times M_i \times C_{d_i}\).
18:   \(C' = C' - (N_i - g) \times M_i \times C_{d_i}\) and \(C' = C' - (N_i - g) \times M_i \times C_{d_i}\).
19: Update lightpath \(p_{s,d,i}\), remove \(p_{s,d,i}\) from \(P\) and add \(p_{s,d,i}\) to \(P^{new}\) (path \(p_{s,d,i}\) with g FSs)
20: else if \((C' < C_i - (g \times M_i \times C_{d_i}))\) then
21:   //partial bandwidth of path \(p_{s,d,i}\) can be contracted to meet the requirement
22:   Reduce the bandwidth of the lightpath \(p_{s,d,i}\) to \(C'_i = \left\lfloor \frac{C_i - C^c}{g} \right\rfloor \times M_i \times C_{d_i}\).
23: Update \(C' = C' - C_p\) and \(C' = C' - C_i\). // terminate algorithm
24: Update lightpath \(p_{s,d,i}\), remove \(p_{s,d,i}\) from \(P\), and add \(p_{s,d,i}\) to \(P^{new}\).
25: end if
26: end while
27: **return** \(P^{new}\) and success.

4. SIMULATION RESULTS

The NSFNET network (shown in Figure 2(c)) was used for simulations. In Figure 2(c), the number nears the line represents the distance of the link and the unit of the distance is kilometers (km). The proposed algorithms were coded by using C++ programming language and the boost graph library (BGL) [23] with version 1.61.0. All simulations were run on a notebook computer with Intel Core i7-4710HQ 2.50 GHz CPU, 16.0 GB RAM and with Windows 10 pro 64-bit operating system. The BW capacity is randomly selected according to a uniform distribution within 12.5–200 Gb/s. The transmission reaches of BPSK, QPSK, 8-QAM, and 16-QAM signals are the same as the [4], and the simulation parameters are summarized in Table 1.

For the simulations, 3000 requests are randomly generated. Each pair of nodes \((s,d) \in V \times V\) has the equal probability to be selected as the source-destination pair. In the set of requests, the first 100 connections are new or BW-increasing connections, after that about a quarter of all connections are with zero BW (all allocated lightpaths for this connection will be deleted) or are new/expanded/contracted connections.
Fig. 2. Contraction example for selected path \( p \) (a) \( C_i > C^c \geq C_i - g \times M_i \times C_{sl} \), (b) \( g \times M_i \times C_{sl} \leq C_c < C_i - g \times M_i \times C_{sl} \), (c) NSFNET.

### Table 1. Simulation Parameters.

<table>
<thead>
<tr>
<th>Name or Notation of Parameter</th>
<th>Setting Value</th>
<th>Name or Notation of Parameter</th>
<th>Setting Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of requests</td>
<td>3000</td>
<td>( B ), number of frequency slot per link</td>
<td>600 or 300</td>
</tr>
<tr>
<td>number of candidate paths (( K ))</td>
<td>5</td>
<td>( C_i ), capacity of a FS with ( M=1 )</td>
<td>6.25 or 12.5GHz</td>
</tr>
<tr>
<td>( N_{GB} ), number of slots for GB per conn.</td>
<td>1</td>
<td>( g ), bandwidth allocation granularity</td>
<td>1, 2, 3, 4</td>
</tr>
<tr>
<td>Transmission reach of BPSK (( M=1 ))</td>
<td>9,000 km</td>
<td>Transmission reach of QPSK (( M=2 ))</td>
<td>4,800 km</td>
</tr>
<tr>
<td>Transmission reach of 8-QAM (( M=3 ))</td>
<td>2,400 km</td>
<td>Transmission reach of 16-QAM (( M=4 ))</td>
<td>1,200 km</td>
</tr>
</tbody>
</table>

### 4.1 Performance Criteria

Several performance criteria are considered in this paper, they are (1) Blocking Probability (BP): BP is defined as the ratio of blocked connection BW versus total request BW. BP is a commonly used metric for assessing the performance of service provisioning algorithms. (2) Fragmentation Ratio (FR): fragmentation is another interesting factor to investigate in dynamic RMSA. The fragmentation ratio of the network is defined as \( FR = \frac{\sum_{e_i \in E} MaxBlock(e_i)}{\sum_{e_i \in E}(B - \text{sum}(e_i))} \). (3) Computation time and (4) total number of active lightpaths.

#### 4.2 Simulations

Six cases of the combined PSPs (described in Section 3.1) are used, and each of which is represented by the form “expansion|contraction”. Thus, the six cases are MaxESF|MaxFRF, MaxESF|MinWHF, SHF|MaxFRF, SHF|MinWHF, MinFRF|MaxFRF and MinFRF|MinWHF. For the parameters \( K = 5 \), \( B = 600 \), \( c_{sl} = 6.25 \), \( g = 1 \) and 3000 requests, the simulation results for different E|C cases are shown in Figure 3. Figure 3(a) shows that the MinFRF|MaxFRF is the quickest method and the SHF|MinWHF is the slowest one. Figure 3(b) shows that the MinFRF|MaxFRF is the best method and the MaxESF|MaxFRF is the worst one, the BR difference between these methods is about 2%. Figure 3(c) shows that the MinFRF|MinWHF and MaxESF|MinWHF can get better FR, and the MaxESF|MaxFRF can get lower FR. Figure 3(d) shows that the MaxESF|MaxFRF will establish more lightpaths than other cases.

To know the efficiency of the proposed SEC scheme, several algorithms are also implemented for comparisons. We use “MP” for multipath routing, “SP” for single-path routing, “SEC” for allowing spectrum extraction/contraction operation, “AD” for only allowing lightpath added/deleted operation for the connection request. That is, for the MP_AD, the BW-varying (expansion/contraction) connection request is treated as follows: first, all current lightpaths are deleted, and then a set of new lightpaths are established by performing the RMSA-HSMR algorithm [4] to meet the new bandwidth
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Fig. 3. Simulation results for 3000 requests with different E[C] PSPs for MP_SEC (for $B=600$, $K = 5$, \( C_{sl} = 6.25 \), $g = 1$): (a) CPU time in seconds, (b) BR, (c) FR, (d) the number of active lightpaths.

Fig. 4. Simulation results for 3000 requests with different methods MP/SP or AD/SEC (for $B=600$, $K = 5$, \( C_{sl} = 6.25 \), $g = 1$): (a) CPU time in seconds, (b) BR, (c) FR, (d) the number of active lightpaths.

requirement. For the SP_SEC, the single-path routing is used but the spectrum expansion/contraction operation is allowed for the connection request. In this method, the PSPs proposed in this paper was not applied since there is only one established lightpath for the connection request. The SP_AD is the traditional single-path RMSA routing method [3] and the SEC operation is not applied.

For the parameters $K = 5$, $B = 600$, \( C_{sl} = 6.25 \), $g = 1$ and 3000 requests, the simulation results for different methods are shown in Figure 4. For the multopath SEC method, the MinFRF|MaxFRF scheme is applied to select candidate path. Figure 4(a) shows that the single-path routing is quicker than multipath-routing methods. Figure 4(b) shows that the MinFRF|MaxFRF is the best method and the SP_AD is the worst one. In general, multipath-routing is better than single-path routing, and SEC methods can reduce BR for both SP and MP methods, the difference between these methods is about 8%. Figure 4(c) shows that the MP_AD can get better FR, and the MinFRF|MaxFRF can get lower FR. Figure 4(d) shows that the MinFRF|MaxFRF can establish more lightpaths than other methods in most of the cases.

The simulation results of 3000 connection requests and different values of $g \in \{1,2,3,4\}$ of the MP_SEC (MinFRF|MaxFRF is used for comparison) and MP_AD are shown in Figure 5. Figure 5(a) shows the computation time of simulations with different values of $g$. The case with $g=1$ is the most time-consuming method. In Figure 5(b), for SEC cases, the case with $g = 2$ can get the lowest BR. For AD cases, the case with $g = 4$ can get the lowest BR. In general, the BR of MP_SEC cases is better than that of the cases in MP_AD. The value of $g$ represents the minimal number of FSs of a lightpath, greater $g$ will cause wasting FSs if the lightpath with smaller ($< g$) required FSs is allocated. Moreover, for the SEC cases, the BR of the case with $g = 2$ is the best one, even better than the case with $g = 1$. This may be the reason that lightpaths with smaller BW (g=1) are allowed to be established may cause the number of guard bands increases and bandwidth
waste. Figure 5(c) shows the FR of the network for simulations with different values of $g$. Cases for MP_SEC can get larger FR than that of the MP_AD. The case with $g = 2$ can get better FR than other cases in both SEC and AD methods. Figure 5(d) shows that cases for MP_SEC can establish more lightpaths than that of the MP_AD.

To know the effectiveness of the $C_{sl}$ and $g$, two cases with $C_{sl} = 6.25$ and 12.5 GHz are combined with two cases with $g=1$ or 2. For the case with $C_{sl} = 6.25$ and 12.5, the value of $B$ is set to 600 and 300, respectively. That is, for the same connection $R(s, d, C)$, the required FSs will be different in these two cases. The required number of FSs for the case with $C_{sl} = 6.25$ may be close the twice of the case with $C_{sl} = 12.5$. The simulation results for case $g = 1$ and $g = 2$ are shown in Figure 6(a)–(d) and Figure 6(e)–(h), respectively.

In Figure 6(a)(e), for the case with same $C_{sl}$ but different $g$, the computation time of the cases are almost the same. In the case with same $g$ and $C_{sl}$, the computation time of the cases with MP_SEC is lower than that of the cases with MP_AD. In Figure 6(b)(f), for the case with same $C_{sl}$ but different $g$, the BR of the cases with $g = 2$ is lower than that of the cases with $g = 1$. In the case with same $g = 1$ but different $C_{sl}$, the BR of the cases with $C_{sl} = 12.5$ is lower than (but close to) that of the cases with $C_{sl} = 6.25$ (shown in Figure 6(b)). In the case with same $g = 2$ but different $C_{sl}$, the BR of the cases with $C_{sl} = 6.25$ is lower than that of the cases with $C_{sl} = 12.5$ (shown in Figure 6(f)).
Figure 6(c)(g), for the case with same $C_{sl}$ but different $g$, the FR of the cases with $g = 2$ is higher than that of the cases with $g = 1$. In the case with same $g = 2$ but different $C_{sl}$, the FR of the cases with $C_{sl} = 12.5$ is lower than that of the cases with $C_{sl} = 6.25$ (shown in Figure 6(g)). In Figure 6(d)(h), for the case with same $C_{sl}$ but different $g$, the number of active lightpaths of the cases with $g = 1$ is smaller than that of the cases with $g = 2$.

5. CONCLUSIONS

In this paper, the spectrum expansion/contraction problem for multipath routing with time-varying traffic on EONs has been studied. For a given EON and a set of connection requests, the goal is to design a spectrum expansion and contraction method to update the CF and the channel size of the lightpath to fit the required of the request. The expansion and contraction algorithms are proposed and several path selecting policies are designed. Simulations show that the multipath routing with SEC operation can get lower blocking probability than traditional multipath routing scheme, and the case with policy MinFRF|MaxFRF can get the best performance within the proposed path selecting policies.

6. ACKNOWLEDGMENTS

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