# On Complete Coverage Path Planning Algorithms for Non-holonomic Mobile Robots: Survey and Challenges

Amna Khan[1], Iram Noreen[2], Zulfiqar Habib[3]
Department of Computer Science, COMSATS Institute of Information Technology, Lahore
[1]amna.cs@gmail.com, [2]iramnoreen@gmail.com, [3]drzhabib@ciitlahore.edu.pk

**Abstract**

*The problem of determining a collision free path within a region is an important area of research in robotics. One significant aspect of this problem is coverage path planning, which is a process to find a path that passes through each reachable position in the desired area. This task is fundamental to many robotic applications such as cleaning, painting, underwater operations, mine sweeping, lawn mowing, agriculture, monitoring, searching, and rescue operations. The total coverage time is significantly influenced by total number of turns, optimization of backtracking sequence, and smoothness in the complete coverage path. There is no comprehensive literature review on backtracking optimization and path smoothing techniques used in complete coverage path planning. Although the problem of coverage path planning has been addressed by many researchers. However, existing state of the art needs to be significantly improved, particularly in terms of accuracy, efficiency, robustness, and optimization. This paper aims to present the latest developments, challenges regarding backtracking sequence optimization, smoothness techniques, limitations of existing approaches, and future research directions.*

**Keywords:** Complete coverage path, non-holonomic, mobile robots, backtracking optimization, path smoothness.

## 1. Introduction

Complete Coverage Path Planning (CCPP) is the problem of finding a path that passes through all the points in the workspace from a starting point to a final point while avoiding obstacles. CCPP is a fundamental problem in robotics with numerous applications in real world such as demining [1] , agriculture and farming [2], cleaning [3, 4], inspection of complex structures [5], seabed mining [6], and underwater operations to name a few. Coverage efficiency of a CCPP algorithm is determined by total coverage ratio, total time required for complete coverage, total path length and energy consumption required to cover the path [3, 7].

Generally, the coverage algorithms are categorized as offline and online algorithms [8]. Offline coverage algorithms use fixed information and environment is known in advance. Complete coverage planned by genetic algorithms, neural networks, cellular decomposition, spanning trees, spiral filling paths and ant colony method falls in this category [4]. Whereas, online coverage algorithms use real time measurements and decisions to sweep the entire target area. In online approaches complete environment map can only be generated by the robot's exploration such as executing an action and observing the consequences of these actions. Sensor based approaches are popular candidate for this category.

In CCPP, two standard basic motions are followed to perform coverage, 1) the square spiral motions, and 2) the boustrophedon (back-and-forth) motion (see Fig. 1). The main advantage of these basic motions is that they can cover region of any shape and can be used as a base for more complex movements particularly in an environment full of obstacles. A CCPP algorithm is complete if the robot sweeps the workplace such that union of all the sub-trajectories completely covers the workplace in finite time. A CCPP algorithm is robust if it is complete and at least one active robot performs the coverage task. A CCPP algorithm is non-overlapping if the robot does not cover the already covered area [9].

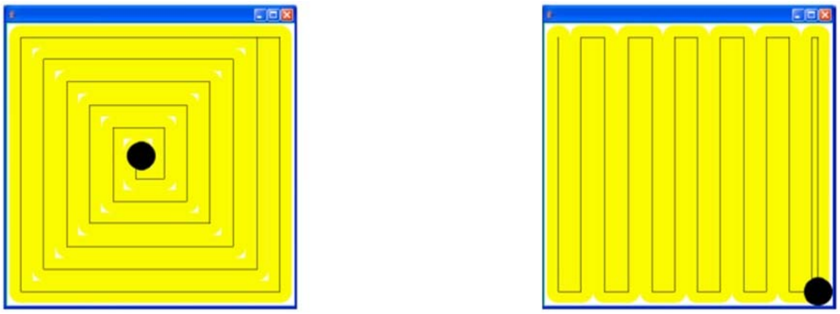


Fig. 1: Spiral Motion (Left), Boustrophedon Motion (Right) [10].

In literature, while performing CCPP three criterion are given importance, 1) the environment decomposition technique, 2) the sweep direction (for reducing total number of turns), and 3) the optimal backtracking mechanism. An environment decomposition technique determines the strategy to divide the environment into smaller regions (cells) for effective coverage. Sweep direction influences the optimality of the generated paths for each sub-region [11]. Within generated trajectory, straight lines take less time than turns because the robot must slow down to make turn [12]. Hence, it is desirable to determine the optimal sweeping direction for each cell separately because decomposition create regions of different shapes and sizes. The sweep direction for a narrow region would be different from the broader region. The motion planning of robot from one small region (cell) to other is achieved using suitable backtracking mechanism. The optimization of coverage sequence reduces completion time consequently increasing efficiency of CCPP. When there is no point left to be backtrack, the coverage is said to be completed.

CCPP can be achieved by using single robot or multirobot coverage according to the size of the environments. Single robot coverage is suitable for the coverage task in relatively small environment such as homes, workplaces and restaurants because of the simplicity of its design and program. The multirobot coverage is appropriate for large environment because the coverage task is performed by dividing the large environment into small sub-regions and covering those sub-regions simultaneously. If one of the robots fails during coverage task, other robots can easily cover the remaining environment.

A CCPP algorithm returns a coverage path that represents a detailed sequence of motion commands for robot in order to perform coverage [12]. Usually, CCPP algorithms generate a linear, piecewise path that composed of only straight lines and sharp turns. These path are not feasible to follow for autonomous underwater vehicles (AUVs), Unmanned Air Vehicles (UAVs) and Unmanned Ground Vehicles (UGVs). In order to make these paths applicable for real time applications in such robots, smoothness must be incorporated in robotic path. Smoothness of the path within the planned coverage can be achieved by first reducing the total number of turns and then smoothing sharp corners while keeping the length of path shortest possible [3].

## 1.1 Motivation

In 2013, Galceran et al. [7] presented a survey on coverage path planning methods with main focus on environment decomposition techniques. However, optimized backtracking and smoothness techniques in context of coverage path planning were not discussed in [7]. At present, to the best of our knowledge, no comprehensive literature review exists on these two important aspects of CCPP, i.e., backtracking sequence optimization and smoothness techniques. The evolutionary algorithms have shown tremendous success in solving complex combinatorial problems such as Hamiltonian cycles, TSP and many more. However, evolutionary algorithms are not explored in previous notable surveys [7, 8]. Therefore, there is a need to investigate advantages, limitations and potential of these algorithms to achieve optimization in real world CCPP problem. Path smoothing techniques constitute an important research area in planning feasible paths for non-holonomic mobile robots and have proven their effectiveness by reducing execution time. There is a need to explore the possibility of integration, restrictions and advantages of these smoothing techniques in CCPP problems. This paper is an effort to fill in this gap for single robot coverage and it provides a discussion on significant smoothness and backtracking techniques in the last five years.

The structure of this paper is organized as follows. The next section includes a brief discussion about major environment decomposition approaches. Section 3 summarizes the most frequently used backtracking sequence optimization techniques. Some common trajectory smoothing methodologies are discussed in Section 4. Section 5 summarizes the current state of the art techniques along with the research contributions and limitations. The challenges are highlighted in Section 6. Finally, concluding remarks and future research directions are given in Section 7.

## 2. Environment Decomposition Approaches

A configuration environment constitutes obstacles, free space and the robot itself. Therefore, the first step towards CCPP is to divide the environment into obstacles and free space configurations. This section summarizes the environment decomposition advancement in recent years to investigate the limitations and advantageous aspects in the current state of the art.

### 2.1 Random Coverage Path Planning

The random coverage path planning methodology is used by several cleaning robots. The main idea behind this approach is to move the cleaning robot in an arbitrary direction in a straight line until it collides with an obstacle. After, collision the cleaning robot turn at some random angle and repeats the straight line motion. This process continues until whole area is covered. Some mobile robots use spiral motion to cover area and follow random motion when obstacles are detected or the end of parameter (such as wall) is found. The coverage performance of several mobile cleaning robots in reduced cleaning environment with limited sensors and memory requirements is discussed in [13]. Liu et al. [14] presented an online novel approach based on random path planning algorithm. Their proposed algorithm is efficient, robust, and flexible for a small unknown environment. However, random coverage path planning is not feasible for coverage in larger environments. A sample random coverage path during cleaning is shown in Fig. 2. Here, 'S' represents starting point of robot, 'C' denotes current point, red circles indicate the collision with obstacle boundaries and 'O' stands for 'obstacles' within environment.

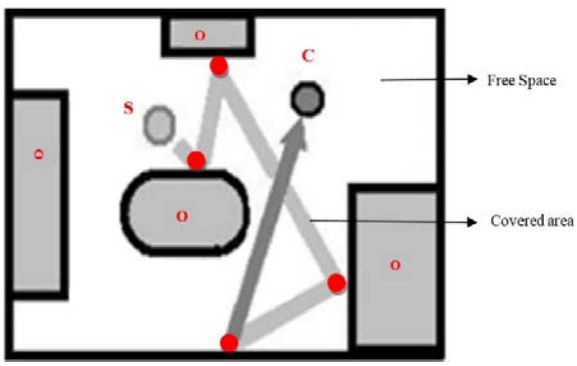



Fig. 2: Random coverage path planning during cleaning [14].

### 2.2 Cell Based Decomposition Techniques

The cell based decomposition techniques are one of the earliest approaches used for CCPP. They decompose free space within the environment into non overlapping regions called 'cells' to perform the coverage. The decomposed environment is represented as an adjacency graph where, cells represent nodes and edges represent a link between two adjacent cells. The nodes of an adjacency graph represent regions to cover. An exhaustive walk through the built adjacency graph can be performed to ensure complete coverage of the desired environment. The coverage of the regions is a local task performed by determining the sweep direction and type of motion for the region. The space within a cell can be easily filled using simple motions such as boustrophedon and square spiral motions. Such type of decomposition is suitable for an offline CCPP.

One of the most classical methods for cell based decomposition is exact cell based decomposition approach. A complete CCPP algorithm of mobile robots using exact cell based decomposition is proposed in [15] which combine local coverage with a global planning approach using boustrophedon motion. An exact cell based decomposition is a simple technique to implement. However, it could result in unnecessary small sub-regions that could be easily avoided by using any other variation of the approach. The exact cell based decomposition can be further extended to trapezoidal and boustrophedon decomposition approaches.

The trapezoidal decomposition is an offline technique and handles only planar and polygonal spaces [16]. Once environment is decomposed a simple boustrophedon motion can be used for coverage of each region. A trapezoidal decomposition based CCPP algorithm for mining robots is discussed in [17]. A major limitation to this approach is the creation of numerous small regions as shown in Fig. 3. Moreover, the complexity of obstacles in real world applications may differ drastically.

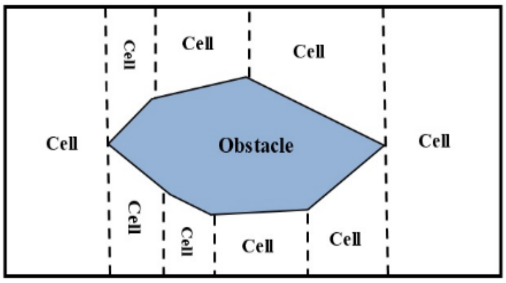



Fig. 3: Trapezoidal Decomposition (Dotted Lines represents sub-regions formed).

The boustrophedon cell decomposition [18] is an extension to the trapezoidal decomposition. It reduces total number of regions formed by the trapezoidal decomposition method by merging all the intermediate cells between two critical points into one cell. The cell formed as a result of merging could be covered by one single continuous motion. The advantage of using this approach is that the robot can now perform coverage in the presence of the curved and circular shaped obstacles as well.

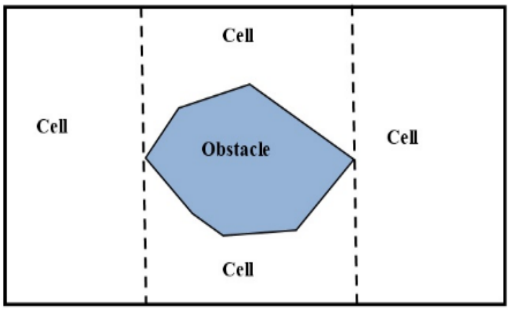



Fig.4: Boustrophedon Decomposition resulting in 4 cells only.

The Morse decomposition [19] technique generalized the boustrophedon method by using Morse functions to determine the critical points for region decomposition. Different cell shapes can be obtained by choosing different Morse functions. Morse decomposition can be applied to any n-dimensional space. The Morse decomposition technique has an advantage of handling non polygonal obstacles along with the polygonal shaped obstacles. The Morse decomposition technique is suitable for coverage with detectors that are the same size of the robot [20].

### 2.3 Grid Based Methods

The grid based methods represent the environment in the form of uniform grid cells. Such methods support easy representation of the environment in memory, resulting in easy robot localization and mapping during coverage task. Voronoi diagrams, distance maps, configuration space maps, and neural network based environment representation are a few extensions that are derived from the grid based representation of the environment [7]. Shivashanker et al. [13] proposed a real time coverage path planning algorithm in unknown environment. However, the approach was incapable of handling dynamic obstacles. Lau et al. [21] presented an efficient grid based workspace decomposition for mobile robots navigation in dynamic environments. The limitation of grid based coverage algorithms include exponential growth in memory requirement with the increase in map size. Moreover, irregular shaped objects have an imprecise representation due to the rigid grid cell structure.

### 2.4 Online CCPP Techniques

It is not always possible to have a prior knowledge of the region for coverage. Robots can perform decomposition of the unknown environment by using incoming sensor data to perform CCPP. This type of coverage is called online or sensor based coverage. A dynamic path planning approach for multirobot coverage considering energy constraints is proposed in [22]. The proposed solution is a modified Ulusoy's partitioning algorithm for agent based robot architecture. The algorithm constructs the sensor-based coverage paths using Generalized Voronoi Diagram (GVD), that account for robot energy capacities both in known and unknown environment [22]. Another sensor based approach using exact cell decomposition for online coverage path planning is discussed in [23]. The proposed algorithm follows an incremental approach for the decomposition of environment into cells. While constructing the map of the workplace robot merges the previous map information with the new data using Hough scan matching. The robot then uses the line features to construct the cell region. Once, whole environment is decomposed, the coverage is achieved by following template based path planning.

### 2.5 Sampling Based Coverage

Deterministic complete path planning algorithms work effectively in an environment where obstacles are already known. However, for dynamic obstacles in an unknown environment the computational time for deterministic and complete algorithms grows exponentially [24]. As an alternative of complete path planning algorithms, sampling based algorithms have gained much attention. Sampling based algorithms enable the development of planning algorithms that are insensitive to the particular dimensions [25]. They can be classified as a single query or multiple query planning algorithms. Rapidly-Exploring Dense Trees (RDTs) and Rapidly-Exploring Random Trees (RRT) are used to develop efficient single query planning algorithms. RRT [26] and its different variations are extensively used in path planning algorithms for finding an optimal path between source and destination. The unique advantage of using RRT is that it can be directly applied to non-holonomic and kinodynamic planning [26]. Conventional RRT satisfies the differential constraints of the system by choosing the allowable input and then applying forward simulation [27]. The RRT quickly expands in a direction of unexplored region. This property of the RRT algorithm can be used in efficiently finding the uncovered area within the coverage terrain. Englot et al. [5] presented a sensor driven, sampling based iterative coverage approach for complex structures using RRT* [28] to shorten the feasible path over complex structure in terms of time and path length.

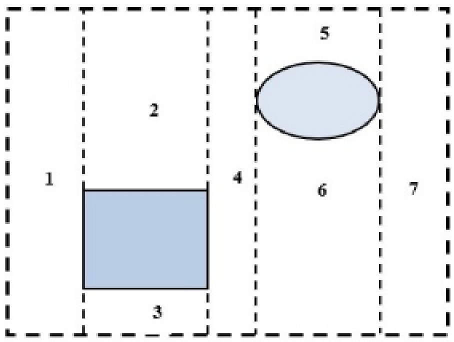
### 2.6 Spanning Tree Coverage (STC)

Gabriely et al. [29] first proposed spiral-STC algorithm with a very basic idea of dividing the environment into grid cells of size twice as that of the robot. Hsu et al. [30] proposed another solution for the optimal complete coverage path planning using an improved spiral motion algorithm with backtracking for the controller input in order to achieve the goals of minimum execution time and human safety. However, the proposed method generated linear path with sharp turns making it non-suitable for non-holonomic robots.

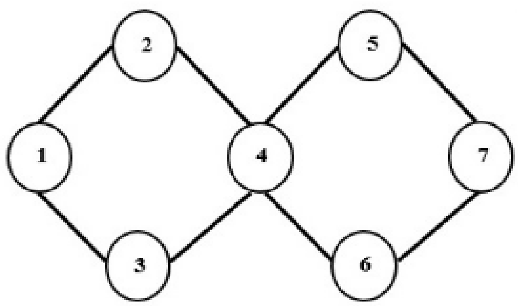
A DFS based spanning tree coverage algorithm is proposed in [31] that covers the unknown environment. However, the algorithm did not consider coverage of partially occupied cells and terrain with narrow openings. Senthilkumar et al. [32] presented another approach for decentralized multirobot based online CCPP using multiple extended spanning trees. The proposed algorithm mimics ant like robots for completing the coverage task.

## 3. Sequence Optimization Techniques

A common approach used to solve CCPP problem is to mimic the classic "Travelling Salesman Problem (TSP)" [11] to cover the sub-regions within a decomposed environment. A common invariant of TSP approach found in literature is to formulate the coverage of environment as the Hamiltonian cycle problem [31]. Therefore, after decomposition of the environment into small blocks for coverage (see Fig. 5 (a)), formation of adjacency graph is the most integral step (see Fig. 5 (b)). Optimal backtracking sequence for coverage is formulated by using the resulting adjacency graph. The most frequently used sequence optimization techniques are discussed briefly in this section.




**(a)**




**(b)**

Fig. 5: (a) Boustrophedon decomposition resulting in 7 cells. (b) Adjacency graph of the environment.

### 3.1 Greedy Algorithms

A greedy algorithm builds up a solution to the problem gradually by always choosing the next choice that offers the most obvious and immediate benefit [33]. A greedy algorithm makes a locally optimal choice that will lead to globally optimal solution through a sequence of choices made at every intermediate step. Greedy algorithms are generally fast. However, they could fail to find the global optimal solution because they do not operate exhaustively on all the data.

***Depth First Search (DFS)***

DFS is an extensively used search technique in graphs. A DFS based optimization of coverage sequence for agriculture robot is presented by Zuo et al. in [34]. However, the obstacles considered during simulation and testing phase were simple as contrary to the complex obstacles in the real agriculture environment. DFS based spanning tree for continuous coverage is proposed in [35]. The proposed algorithm uses an improved

STC based algorithm to optimize the total number of U-turns within each sub-region. Moreover, coverage efficiency was enhanced by enabling shifting of the mowing direction in each sub region. Jin et al. [2] presented an optimal solution for coverage in agriculture field using DFS. However, the proposed algorithm only works for planar fields with no obstacles. Moreover, the model did not consider the cost of turning between two edges. Paratama et al. [36] proposed a DFS based efficient coverage algorithm for underwater mining robots. The proposed strategy effectively reduced overlapping paths and the total number of turns.

***Dijkstra's Algorithm***

The Dijkstra's algorithm [37] is a graph based greedy algorithm that is widely used in solving single source shortest path problems. It works by visiting vertices within a graph starting from the 'source' vertex and moving towards the 'goal' vertex by assigning distance cost to each neighbor vertex. The vertex with the smallest cost is then selected as a new vertex the process is repeated until we reach the 'goal' vertex. An efficient CCPP algorithm for cleaning robots is presented in [10] that uses an extension of boustrophedon cellular decomposition technique combined with simple Dijkstra's algorithm for sequence optimization. The Dijkstra's algorithm only handles static obstacles within an environment. Moreover, it is inefficient with respect to memory storage requirements.

***A* Algorithm***

The A* algorithm [38] is an extension to the Dijkstra's algorithm. The A* algorithm chooses a neighbor vertex by considering the cost of the function along with a heuristic function that helps avoid unnecessary exploration to the neighbor vertices. A computationally low cost and efficient approach for online CCPP in an unknown environment for cleaning robots based on boustrophedon motions and A* algorithm is presented in [4]. The robot performs a single boustrophedon motion for coverage in an unvisited node until it reaches a critical point. The A* algorithm is used to decide the best backtracking point for coverage. The coverage is completed when no backtracking point is found. The proposed algorithm works effectively in unknown environments with arbitrary shaped obstacles. Furthermore, the proposed algorithm was efficient with respect to the coverage time, the coverage path length, the total number of boustrophedon motions and small changes in heading angle [4]. Another efficient solution for online global path planning strategy is proposed in [39]. The limitation of this approach is that the generated path is composed of straight line segments. Moreover, the current algorithm is not capable of handling a complex environment with moving obstacles. An algorithm to accelerate the processing time for CCPP by combining two basic algorithms A* and Time Varying Environment (TVE) [40] for the seabed gliders is proposed in [41]. The algorithm was memory efficient, but it did not include energy constraints of the non-holonomic robot. The A* algorithm is memory efficient; however, it cannot handle the dynamic environment constraints.

***D* Algorithm***

A. Stenz [42] proposed an optimal and efficient path planning algorithm for handling dynamic unknown environment called D*. The D* algorithm makes an assumption about the unknown environment that it does not contain an obstacle and start traversing from the 'source' position. When an obstacle is detected, the information of the environment map is updated and so is the shortest path calculation.

Dakulovic et al. presented an extension of the D* search algorithm in [43] called Complete Coverage D* algorithm (CCD*). The main contribution of this coverage algorithm was taking into consideration the dimension of the robot for the floor cleaning problem. Dakulovic et al. further extended the CCD* algorithm in [1] to find the coverage path for demining robot in an unknown environment. Their proposed algorithm considered the static as well as the dynamic obstacles. However, the limitation of the approach was that few regions remain unvisited due to non-perfect path following and frequent changes in the path direction (see Fig. 6). Moreover, the generated path consists of unnecessary singularities making it unfeasible for robot following. Three snapshots of the simulation of proposed approach for replanning steps are shown in

Fig. 6. The green area indicates covered area. The red dotted line shows the path of the robot and the blue solid lines show the path of the tool. The bold curves show the driven trajectory [1].

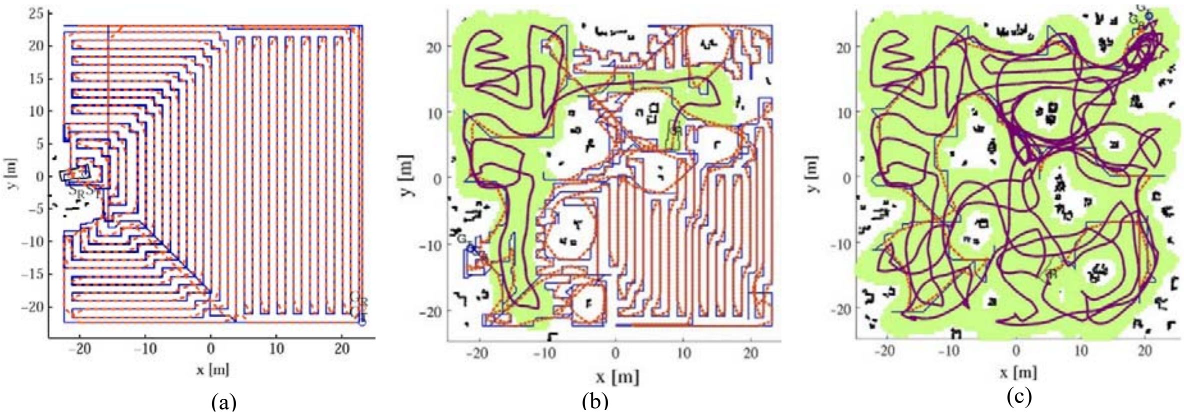



Fig. 6 : (a) Initial planning phase of coverage using CCD*. (b) Replanning phase along with partial covered area. (c) Final coverage using CCD* [1].

***Theta* Algorithm***

All the backtracking path planning algorithms discussed so far generate a path that is constrained to grid edges. However, the theta* [44] search algorithm has flexibility to propagate information along grid edges without constraining their paths to grid edges. The theta* is a variant of the A* search for any angle path planning. In A* algorithm, parent of the vertex should be a neighbor vertex whereas, in theta* the parent of the vertex could be any vertex as long as it exists in the line-of-sight of the vertex. Theta* is simple, fast, and generates short and more realistic looking paths [44]. Viet at al. [45] presented an online CCP approach for a team of mobile robots in unknown environment. The author used variation of theta* for performing backtracking for multiple robots combined with boustrophedon motion. However, dynamic environment constraints for mobile robot were not considered.

### 3.2 Dynamic Programming

Dynamic Programming (DP) is the process of breaking a complex problem into a collection of simple sub problems that exhibit the properties of overlapping sub problems and optimal sub structure. It solves the repeated calculation of sub-problems by storing solutions of similar sub problems. The steps followed while developing a solution using dynamic program as discussed in [46] are:

1. Characterize the structure of an optimal solution.
2. Recursively define the value of an optimal solution.
3. Compute the value of an optimal solution, typically in a bottom-up fashion.
4. Construct an optimal solution from computed information.

A dynamic programming formulation can be used to find the optimal sequence of visiting nodes in an adjacency graph of a decomposed environment [12]. A CCPP algorithm that combines local space coverage with global planning using DP is proposed in [15]. However, the proposed algorithm only considered static obstacles within the environment. Moreover, the algorithm used for sub-regions formation resulted in unnecessary small area sub-regions. Another approach presented in [6] used dynamic programming and TSP reduction to generate a coverage path for a known seabed environment. However, the proposed strategy failed to address an unknown environment with dynamic obstacles.

## 3.3 Evolutionary algorithms

The Evolutionary Algorithms (EA) are based on principle of natural evolution, such as biological inheritance and natural selection. An EA randomly selects a candidate set of solutions and apply the quality function as an abstract fitness measure [47]. The evolutionary algorithms tend to find optimal solution by converging from the initial state to the global optimal with the help of a fitness function. A variety of evolutionary algorithms are reported in literature. Some of the most common techniques used in CCPP are discussed briefly here.

***Genetic algorithm***

A Genetic Algorithm (GA) is a heuristic based stochastic algorithm inspired by the idea of natural selection and mutation for solving optimization and search problems [48]. The basic building blocks for GA are mutation, fitness function, selection, and crossover operations. Chromosomes constitute the candidate solution sets. A population set is randomly initialized at first. Later, it is slowly converged to the optimal solution by using mutation, fitness function, selection, and crossover operations.

The GA is extensively used to solve the coverage sequence optimization problem in CCPP [49, 50, 32, 31, 51]. Jimenez et al. [49] presented a novel genetic path planner for sequence optimization based on two basic templates. The approach works well for the static single robot environment. An online algorithm for coverage sequence optimization in CCPP using GA for complex fields is proposed in [51]. The algorithm combines the general practices used to perform coverage with the intelligent support for obstacle avoidance to perform effective coverage with minimum execution time. Another pattern based genetic algorithm approach for optimization of backtracking sequence is presented in [52]. The proposed algorithm enables multiple robots to perform the cleaning task in an unknown environment with unknown obstacles. However, the dynamic environment constraints such as moving obstacles and finding the optimum starting position of robots still remain an open issue. Kapanoglu et al. [53] presented another pattern based GA for multirobot CCPP. The presented GA partition coverage area among multiple robots such that no overlapping occurs, thus resulting in minimum execution time. Hameed et al. [54] discussed an energy efficient CCPP approach for 3-D terrains using GA. The proposed algorithm optimized the driving angle followed by the track sequence optimization resulting in minimum execution time.

***Ant Colony Optimization (ACO)***

An Ant Colony Optimization (ACO) [55] is a probabilistic technique for solving computational problems. An ACO is inspired by the behavior of real ant colonies and was initially applied for solving travelling salesman problem. Zhou et al. [56] presented an extended ACO algorithm to perform an optimized coverage sequence for field operations with multiple obstacles. The algorithm is efficient and feasible for real world application deployment. Chaari et al. [48] presented a hybrid approach using ACO and GA for solving global optimization path planning problem. The proposed hybrid algorithm tries to find the optimal path by using improved ACO and genetic algorithms for a static environment. The use of genetic algorithm ensures minimization of the risk of falling in local minimum by exploring different search spaces. Chibin et al. [57] presented an ACO based algorithm for efficient and optimal sequence coverage based on a distance matrix. The presented algorithm not only covers the entire area but also ensures the minimum planning path.

***Particle Swarm Optimization (PSO)***

The Particle Swarm Optimization (PSO) [58] is a population based stochastic algorithm inspired from the social behavior patterns of organisms living in large groups. PSO is used for optimizing continuous nonlinear functions and performs a parallel search on a space of solutions. In the classical PSO, each particle not only knows its position, velocity, and an associated value, but also its neighbor’s positions and associated values, and the best achieved position.

A PSO based online CCPP algorithm is presented in [3] that is based on the high resolution grid and successfully finds a smooth path with minimal cost for coverage. Pessin et al. [59] compared PSO and ACO for garbage collection and recycling problem in a multirobot scenario. PSO performed well in the respective scenario due to its quick convergence than ACO. Another PSO based partial search algorithm is proposed in [60], exhibiting better results in terms of time efficiency than classic PSO and GA in solving TSP. PSO is also used with other EAs for faster convergence. Two PSO variants, Fuzzy Ant Supervised by PSO (Fuzzy-AS-PSO) and Simplified Ant Supervised by PSO (S-AS-PSO) are proposed in [61] to solve ACO parameter adjustment problem. The proposed algorithms showed good performance on solving the TSP problem than existing methods.

Backtracking sequence optimization techniques frequently used in literature are summarized in Table 1.

Table 1: Overview of backtracking techniques (2010-2015).

| **Sr #** | **Backtracking Techniques** | **Online/ Offline** | **Advantages** | **Limitations** |
|---|---|---|---|---|
| 1. | DFS [34], [35], [2], | Offline | • Requires less memory than trees.<br>• May search from multiple sources. | • Not suitable for online applications because DFS can be stuck in exploring long infinite paths.<br>• There is no possibility of getting the minimal solution if more than one solution exists |
| 2. | Dijkstra's Algorithm [10] | Offline | • Easy to understand and implement. | • Large memory space is required to store and process graph for shortest path.<br>• Cannot be used when the environment is changing dynamically. |
| 3. | A* and its variations [4], [9], [39], [41] | Offline/ Partially known environments | • Reduces search space required as compared to Dijkstra's algorithm. | • A* cannot handle dynamically changing environment, however, some of its variations like greedy A* [9] are capable of handling dynamic environments. |
| 4. | D*,<br>CCD* [43], [1] | Online | • Generates less costly path than grid based algorithms. Capable of handling dynamic environment.<br>• CCD* however, takes into account the shape and size of the robot as well, so more suitable for generating complete coverage paths. | • They can still generate unnecessary longer paths for robots to follow. |
| 5. | Theta* [45] | Offline | • Theta* is simple, fast, and generates short and more feasible paths for robot navigation. | • Cannot handle dynamic environment. |
| 6. | Dynamic Programming [15], [6] | Offline | • Stores previous calculated values in order to avoid multiple calculations. It guarantees optimal solution. | • Expensive in terms of memory requirements. |
| 7. | Genetic Algorithms [32], [49], [51], [52], [53], [54] | Online | • The concept is easy to understand and supports multiobjective optimization. | • It requires a mathematical model of the problem. Encoding depends on the problem heavily.<br>• Moreover, complexity of the model increases with the increase in population size of the samples. |
| 8. | ACO [48], [56], [57] | Online | • ACO is capable of finding the optimal solution even when the | • Speed of convergence to the optimal solution is unknown. |

| | | | | |
|---|---|---|---|---|
| | | | structure of a graph changes at run time. | • Performance is problem specific. |
| 9. | PSO [3], [59], [60], [61] | Online | • Easy to implement.<br>• Achieve global optimal solutions with high probability | • Particles may converge prematurely and cause stagnation. |

# 4 Path Smoothing

A mobile robot comes with the constraints such as bound on its curvature, velocity and acceleration. These constraints restrict the movement of the mobile robot at sharp turns while following the linear piecewise trajectory generated by a CCPP algorithm. While following a trajectory, sharp turns in the generated path cause jerks resulting in discontinuity along the trajectory. Thus, the trajectory cannot be driven at a constant speed. A smooth path enables the robot to follow the trajectory without stopping, slowing down and reorienting on sharper turns. According to Farin [62], smooth path must be free from unwanted singularities (loops and cusps), inflection points, and curvature extrema (if speed of the robot is required to be fast). Most of the present CCPP algorithms generate a path with sharp turns resulting in inefficient movement of the non-holonomic mobile robots, extra fuel consumption, extra working time, and premature damage of robot parts.

The coverage efficiency is the highest concern in CCPP algorithms. The coverage efficiency decreases with an increase in the total operational time [2]. The performance of a CCPP algorithm can be significantly improved by reducing the overlapping paths and smoothness [3]. Maintaining smoothness throughout the coverage path helps to decrease the total operational time of the CCPP algorithm. There are two classes of smooth CCPP in coverage literature; the graphical methods and the function based methods.

## 4.1 Graphical Methods

In graphical method simple shapes like circles, arcs and lines are used to generate a smooth path. Shape based path formation approaches are discussed in [2, 11, 63, 64] for the agriculture operations. Keeping in view the differential constraints of agriculture robot Jin et al. [2] classified the headland turning types into five different categories namely flat turn, ‘U’ turn, bulb turn, hook turn, and minimum headland width turning (see Fig.7). The generated trajectory is compatible with the constraints of the robot; however, it requires complex calculations to create the required shape for turning.

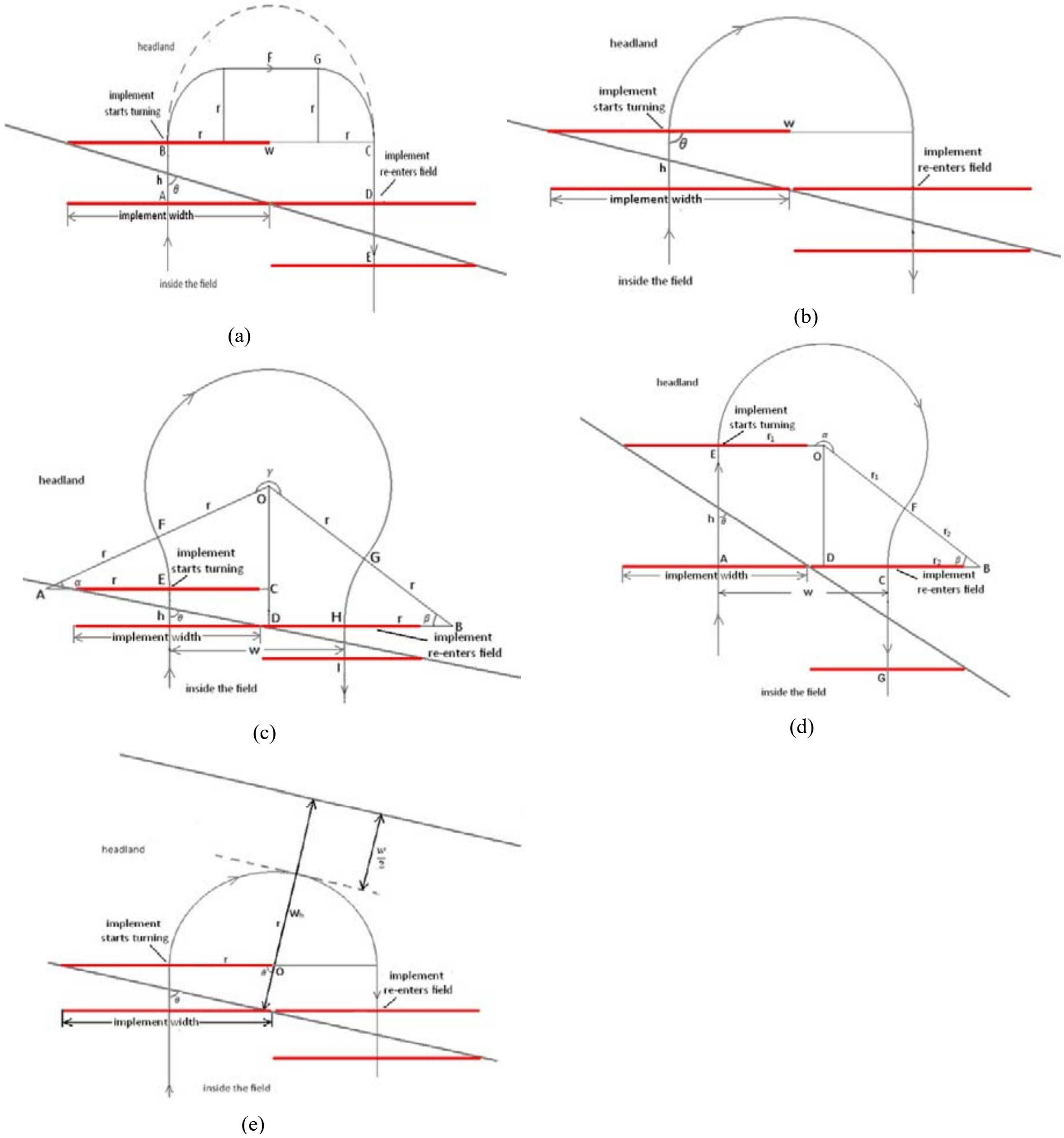



Fig. 7: Different Headland turning shapes as discussed in [2] (a) Flat turn. (b) 'U' turn. (c) Bulb turn. (d) Hook turn. (e) Minimum headland width turning.

Another shape based approach for agriculture vehicle is discussed in [64]. The author identified three types of turns depending upon the kinematic restrictions of the machine and the available space on the headland area. The classified geometric types are π-turn (also referred as U-Turn), **Ω**-turn and T-turn (also called as fishtail turn). Bochtis et al. [65] and Spekken et al. [66] used two basic geometric shapes U-turn and **Ω**-turn for smoothing headland turns and thus reducing the total coverage time of the CCPP. The three most common occurring graphical shapes are shown in Fig. 8.

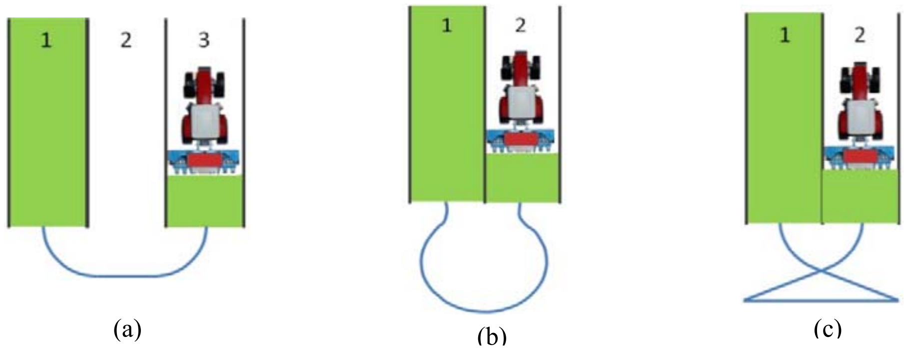



Fig. 8: (a) ∏-turn (U-turn) (b) Ω –turn (c) T-turn (Fishtail-turn) [64]

Another graphical approach used for generating smooth CCPP path of a UAV is discussed in [63]. In order to avoid deviation from the planned trajectory, a circular orbit (curlicue) is formed at each turning point of the UAV (see Fig. 9). However, such a solution results in increased execution time, extra fuel consumption and additional distance travelled.

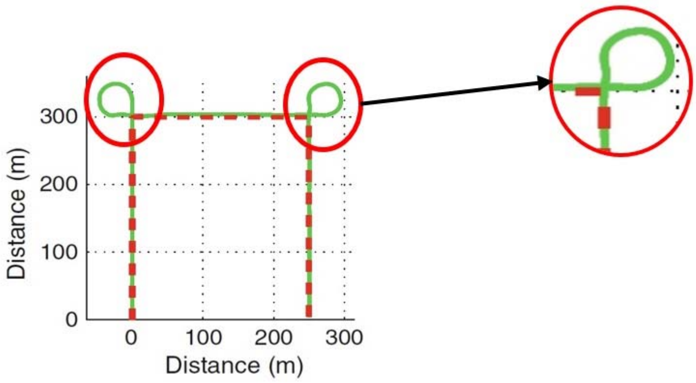


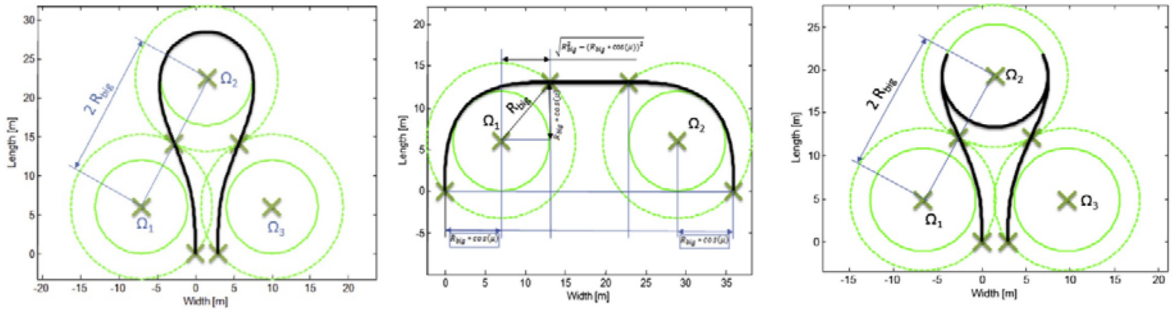

Fig. 9: Generation of Circular trajectory at turning points for coverage efficiency in [63]

Dubin's curve [67] is a traditional approach used in the smooth trajectory generation using segments of line and arc. However, the presence of discontinuities at the joining point of the lines and arcs make it unfeasible for real world applications. Backman et al. [68] presented an extended curvature continuous method using Dubin's curve for agriculture vehicles. The algorithm iteratively generates path segments using Dubin's method and integrates spiral segments to avoid discontinuity at the joining point. Thus, making trajectory easy to follow by real-world robots. Moreover, the algorithm is time efficient as well. Yu et al. proposed Dubin's vehicle based efficient coverage algorithm for agriculture field in [69]. The authors presented a strategy for reducing total coverage time by wisely dividing the environment and then planning coverage path considering non-holonomic constraints of the Dubin's vehicle.

Although graphical methods serve the purpose of smooth trajectory generation for real-world robots. However, they are incompatible with today's CAD/CAM software applications. Moreover, intensive calculations for required shape generation makes them unfeasible for real-world application deployment.

## 4.2 Function Based Methods

In function based methods path trajectory is represented by function equations such as spirals, clothoids, spline based functions, and Bézier functions. Clothoid is one of the simplest and powerful method for smoothing the sharp corners. In order to ensure curvature continuity in trajectory Cariou et al. [70] used elementary primitives (line and a circular arc) connected together with clothoid segments. However, trigonometric and inverse trigonometric functions are used to determine the headland turning type making it computationally expensive. Sabelhaus et al. [71] presented a Curvature Continuous (CC) path approach by using clothoid segments to generate a trajectory for an autonomous steering vehicle (see Fig.10). However, complex computations and non-polynomial approximation functions involved in generating clothoid segments for trajectory make it unfeasible for practical deployment.



(a) (b) (c)

Fig. 10: Smooth path trajectory generation using CC-path (a) Ω-turn (b) U-turn (c) Fishtail turn [71].

Bézier curve is one of the most fundamental modeling tools used in CAD/CAM systems today. A smooth path trajectory for CCPP is presented in [72], where maximum curvature constraint of a non-holonomic mobile robot was considered to generate the coverage path. However, the coverage path overlapping and missing collision avoidance strategy makes it inappropriate for real world applications. A better online complete path coverage and smoothing algorithm was proposed in [3] where, the Bézier curve approximation was used for smoothing the square spiral coverage path. The smoothness introduced by Bézier curve approximation lead to fast coverage and less energy consumption. The simulated results as in [3] are shown in Fig.11.

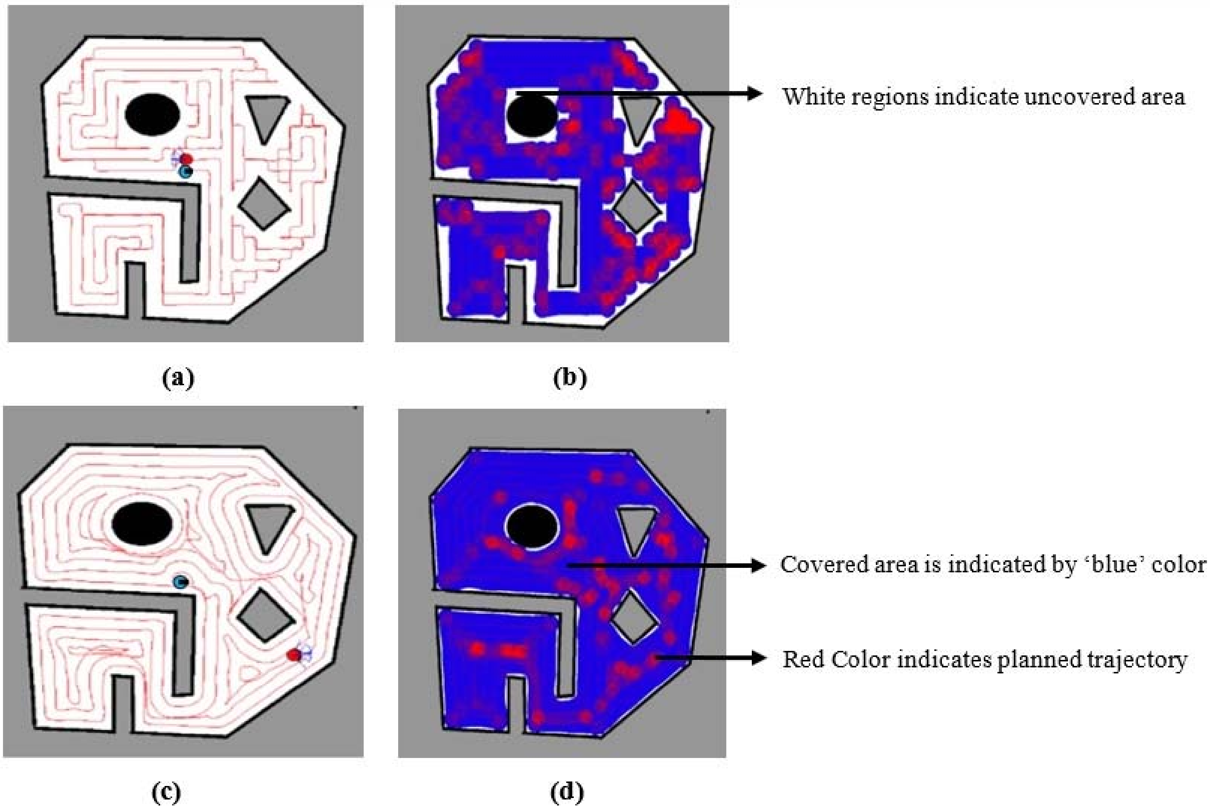



Fig. 11: (a) Path trajectory generated by basic STC. (b) Area coverage performed by following trajectory in (a). (c) Path trajectory generated by using approach in [3]. (d) Area coverage performed by following trajectory in (c).

## 5. State of the art (2010-2015)

The most relevant papers reviewed in this article, along with the research contributions and limitations are summarized below. Table 2 comprises both CCPP approaches and path smoothing approaches.

Table 2: Summary of state of the art (2010-2015).

| Sr # | Author, year | Methodology Used | Application Domain | Research Contributions | Limitations |
|---|---|---|---|---|---|
| 1. | J. Jin and L. Tang, 2010 [2] | Trapezoidal decomposition, Boustrophedon motion and Shape based headland turning type. | • Agriculture Field Coverage | • Optimal coverage path planning for agriculture fields.<br>• Considered dynamic constraints of the farming vehicles. | • Complex trigonometric calculations for determining headland turning angles.<br>• Does not ensure continuity. |
| 2. | I.A. Hameed et al. 2011 [64] | B-patterns, Boustrophedon motion, shape based headline turning for traversing between two edges, and GA | • Agriculture Field Coverage | • Optimization of track sequence and driving angles.<br>• Considers dynamic constraint. | • Continuity is not ensured.<br>• Involves trigonometric and inverse trigonometric functions for determining turning shapes and driving angles. |
| 3. | T. K. Lee et al., 2011 [3] | Spiral path following, virtual wall tracking and following, quintic | • Generalized Approach | • First attempt for path smoothing in CCPP.<br>• Smooth CCPP resulted in faster coverage. | • Computationally expensive.<br>• Required external memory.<br>• Localization issues were not considered. |

| | | | | | |
|---|---|---|---|---|---|
| | | Bézier curve approximation. | | • Less energy consumption. | |
| 4. | M. Waanders, 2011 [10] | Variation of Dijkstra's Algorithm and Boustrophedon Motion. | • Cleaning Task Coverage | • An online coverage path planning algorithm for known environment with simpler dynamic obstacles. | • Dynamic obstacles handling for unknown environment remain an open issue. |
| 5. | Dakolovic et al., 2011 [43] | Path Transform Methodology, D* | • Cleaning Task Coverage | • A CCD* algorithm for cleaning robots handling dynamic obstacles within an environment. | • Kinodynamic constraints of robot were not considered resulting in inefficient trajectory following. |
| 6. | P. Zhou et.al., 2012 [15] | Boustrophedon decomposition, Dynamic programming. | • Generalized Approach | • Boustrophedon unit decomposition was used to divide the environment into small sub regions.<br>• A cost distance matrix was created using local space dividing, sub-space connecting sequence and sub-space walking route.<br>• Dynamic programming was used to find the optimal coverage sequence of the environment. | • Works with known environment with static obstacles. |
| 7. | Kapanoglu et al., 2012 [53] | Online Sensor based coverage, pattern based GA for CCPP | • Generalized Approach | • GA for optimizing sequence in multirobot scenario was used to minimize execution time. | • The multiobjective environment was not considered. |
| 8. | M. Dakulovic and I. Petrovic, 2012 [1] | Complete Coverage D* (CCD*) algorithm. | • Humanitarian Demining | • CCD* algorithm was used to find the coverage path for the static and dynamic obstacles. | • Few regions remain unvisited due to imperfect path following and frequent changes in the path direction. |
| 9. | I.A. Hameed et al., 2013 [51] | Cell decomposition, Genetic algorithm. | • Agriculture Field Coverage | • Cell decomposition was used to divide the agriculture field for CCPP. Genetic algorithm was used to generate an optimal coverage sequence. | • Approach lacks in handling dynamic obstacles. |
| 10. | Spekken et al., 2013 [66] | Coverage sequence optimization technique using Ω-turn and U-turn for CCPP. | • Agriculture Field Coverage | • Graphical shapes are used to incorporate smoothness in the trajectory of an agricultural vehicle. | • The shape optimization method requires high computation cost.<br>• Obstacle avoidance was not addressed. |
| 11. | M. Morin et al., 2013 [6] | Cellular Decomposition, Dynamic Programming, TSP reduction. | • Seabed Coverage. | • An offline seabed coverage approach.<br>• Time efficient. | • Only for obstacle free environment. |
| 12. | Sabelhaus et al., 2013 [71] | Curvature continuous clothoids for CCPP. | • Agriculture Field Coverage | • Curvature continuous clothoids for agricultural vehicles. | • Complex calculations.<br>• Deals with static obstacles only |

| | | | | | |
|---|---|---|---|---|---|
| 13. | A. Janchiv et al., 2013 [73] | Exact cell decomposition, Templates, Flow Network. | • Cleaning Task Coverage | • Static environment with predefined obstacle shapes as templates was considered. | • Target environment contains complex shaped obstacles that cannot be effectively represented by using exact cell decomposition.<br>• Dynamic obstacles were not considered. |
| 14. | H. H. Viet et al., 2013 [45] | Boustrophedon Motion and theta*. | • Cleaning Task Coverage | • CCP algorithm for unknown environment. | • Dynamic obstacles were not considered. |
| 15. | H. H. Viet et al., 2013 [4] | Incremental Boustrophedon motion and A* algorithm. | • Cleaning Task Coverage | • Online complete CCPP of an autonomous cleaning robot capable of working in unknown environment. | • A* was in-efficient with respect to time complexity. |
| 16. | K. Zhou et al., 2014 [56] | Cell decomposition, Ant Colony Optimization. | • Agriculture Field Coverage | • Feasible area coverage plan for agriculture field was developed using a block formation.<br>• Block generation was created in such a way that there were no obstacles within a block so that the robot could work efficiently. | • The approach works only for static environment. |
| 17. | A. Yazici et al., 2014 [22] | GVD, Capacitated Arc Routing Problem (CARP). | • Generalized Approach | • Multi-robot based architecture designed for partially known environment keeping energy constraints into account. | • Computationally complex architecture. |
| 18. | Hsu et.al., 2014 [30] | Spiral motion for CCPP with improved backtracking approach. | • Generalized Approach | • Optimal CCPP for a mobile robot with minimum working time, minimum energy consumption, and mixed operation modes.<br>• Capable of handling dynamic obstacles. | • Generated path is linear and hence, not suitable for non-holonomic mobile robots. |
| 19. | D. H. Kim et al., 2014 [17] | Trapezoidal Decomposition, Special Spanning Trees. | • Mining Application. | • Coverage path planning for a mining robot was proposed for the already known environment. | • Failed to handle dynamic obstacles. |
| 20. | Xu et al., 2014 [63] | Boustrophedon decomposition, Curlicue | • Arial Coverage | • Arial coverage path smoothing using circular orbits (curlicue) at the turning points. | • Complex computations involve in determining the circular orbit for turning. |
| 21. | H. H. Viet et al., 2014 [9]. | Boustrophedon Decomposition, Greedy A* Search. | • Generalized Approach | • Online multirobot coverage approach in unknown environment.<br>• The approach is efficient in terms of the coverage rate, the coverage path length, and the balance of the workloads of the robots. | • Cannot handle dynamic obstacles within an environment. |

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|---|---|---|---|---|---|
| 22. | Yu et al., 2015 [69] | B-Patterns, Non-holonomic constraints of vehicle | • Agriculture Field Coverage. | • Offline approach for efficient sequence optimization reducing overlapping between different swaths | • Obstacle avoidance is not discussed. |
| 23. | Paratama et al., 2015 [36] | Morse Decomposition, DFS. | • Underwater Mining. | • An efficient offline seabed coverage approach.<br>• Used DFS for backtracking sequence optimization. | • Cannot handle dynamic obstacles. |
| 24. | Backman et al., 2015 [68]. | Extended Dubin's curve with spiral segment for curvature continuous path. | • Agriculture Field Coverage | • A curvature continuous path for agriculture vehicles. | • High computational cost.<br>• Can only handle static environment. |

## 6. Challenges

The existing state of the art needs to be significantly improved, particularly in terms of accuracy, efficiency, robustness, and optimization. Achieving optimization is one of the interesting combinatorial problems. A variety of evolutionary algorithms are used to solve very complex combinatorial problems. However, very few addresses the problem of sequence optimization within CCPP, such as genetic algorithm and its variations. There is a need of comparative analysis between different evolutionary algorithms that have already shown their efficiency in solving complex combinatorial problems to solve coverage sequence optimization problem.

Online CCPP approaches provide system with much more flexibility and robustness. However, such approaches use the sensor based information for the environment map generation, thus requiring powerful CPU and auxiliary memory [74]. Moreover, localization errors result in accumulated drift causing uncertainty of the robot's pose. Aforementioned requirements have a direct influence on power consumption of the robot, which is a critical constraint in some applications of CCPP such as aerial coverage. The current state of the art online CCPP algorithms are computationally expensive and may require the use of auxiliary memory and cache or both, when applied to the low power computational environment. Therefore, online CCPP algorithms need to be significantly improved in terms of memory requirement and energy consumption during coverage task. Moreover, incorporating uncertainty in forthcoming location estimations can considerably increase coverage performance.

In recent years, probabilistic sampling based algorithms are extensively used in path planning algorithms. Low computational cost, suitability to address higher dimensional problems and better pragmatic success rate are major benefits of sampling based algorithms. However, very little work like Englot et al. [5] is reported in literature for using sampling based algorithms in CCPP. Therefore, exploiting sampling based techniques in CCPP for unprecedented complexity remains an open research area.

Coverage path planning in a dynamic environment is considerably more difficult than static environment, since simultaneous replanning of coverage path is required when a moving obstacle is encountered. The situation becomes more complex with assimilation of non-holonomic constraints. Therefore, current state of the art coverage algorithms need enhancement for an efficient dynamic environment coverage.

An interesting research problem is incorporating smoothness in the linear path generated by CCPP algorithm. Recent state of the art CCPP smoothing algorithms have either used primitive graphical shapes or complex mathematical functions to generate feasible trajectory for non-holonomic mobile robot. Spline based interpolation functions have proven their efficiency and feasibility for real world applications in path planning algorithms. There still remains an open research issue for incorporating the spline based functions in CCPP algorithms and testing their feasibility for coverage problem. Thus, research challenges in CCPP strive for better potential solutions.

## 7. Conclusion and Future Directions

Complete coverage path planning is an active area of research due to numerous applications. This paper presents an overview of the most recent sequence optimization and path smoothing techniques used in CCPP with a brief summary of challenges in this domain. A considerable research has been conducted to improve efficiency of complete coverage path planning algorithms. Optimization of backtracking sequence and smoothness integration in coverage can notably improve efficiency of CCPP algorithms. Sampling based strategies have proven to be highly successful in path planning. There is a need to explore these strategies for efficient CCPP. Moreover, integrating uncertainty while performing coverage in dynamic environment remains an open issue to be addressed. Smoothness techniques for path planning of non-holonomic robot are under discussion for many years. Recently, citations are reported on smoothness in CCPP. However, the current state of the art CCPP smoothness techniques are either graphical methods or computationally complex functional methods. Incorporating spline based interpolation functions for an efficient coverage path planning algorithm remains an open research problem.


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## Bibliography


[1] M. Dakulovic and I. Petrovic, "Complete coverage path planning of mobile robots for humanitarian demining," *Industrial Robot: An International Journal,* vol. 39, no. 5, pp. 484-493, 2012.

[2] J. Jin and L. Tang, "Optimal Coverage Path Planning for Arable Farming on 2D Surfaces," *Transactions of the ASABE,* vol. 53, no. 1, pp. 283-295, 2010.

[3] T. K. Lee, S. H. Baek, Y. H. Choi and S. Y. Oh, "Smooth coverage path planning and control of mobile robots based on high resolution grid map representation," *Robotics and Autonomous Systems,* vol. 59, pp. 801-812, 2011.

[4] H. H. Viet, V. H. Dang, M. N. U. Laskar and T. C. Chung, "BA* : an online complete coverage algorithm for cleaning robots," *The International Journal of Applied Intelligence, Neural Networks and Complex Problem Solving Technologies,* vol. 39, p. 217–235, 2013.

[5] B. Englot and F. S. Hover, "Sampling-Based Coverage Path Planning for Inspection of Complex Structures," in *Proceedings of the Twenty-Second International Conference on Automated Planning and Scheduling*, 2012.

[6] M. Morin, I. Abi-Zeid, Y. Petillot and C. G. Quimper, "A Hybrid Algorithm for Coverage Path Planning With Imperfect Sensors," in *International Conference on Robotics and Systems, IEEE*, 2013.

[7] E. Galceran and M. Carreras, "A survey on coverage path planning for robotics," *Robots and Autonomous Systems,* vol. 61, pp. 1258-1276, 2013.

[8] H. Choset, "Coverage for robotics - A survey for recent results," *Annals of Mathematic and Artificial Intelligence,* vol. 31, pp. 13-126, 2001.

[9] H. H. Viet, V. H. Dang and S. Y. Choi, "BoB: an online coverage approach for multi-robot systems," *Applied Intelligence, In Press,* 2015.

[10] M. Waanders, "Coverage path planning for mobile cleaning robots," in *15th Twente Student Conference on IT*, Enschede, 2011.

[11] T. M. Driscoll, *Complete coverage path planning in an agricultural environment,* 2011.

[12] W. H. Huang, "Optimal Line-sweep-based Decompositions for Coverage Algorithm," in *Proceedings of the 2001 IEEE International Conference on Robotics & Automation*, Seoul, Korea, 2001.

[13] V. Shivashanker, R. Jain, U. Kuter and D. Nau, "Real time planning for covering an initially unknown spatial environment," in *Proceedings of the Twenty-Fourth International Florida Artificial Intelligence Research Society Conference*, 2011.

[14] Y. Liu, X. Lin and S. Zho, "Combined Coverage Path Planning for Autonomous Cleaning Robots in Unstructured Environments," in *Proceedings of the 7th World Congress on Intelligent Control and Automation*, China, 2008.

[15] P. Zhou, Z. Wang, Z. Li and Y. Li, "Complete Coverage Path Planning of Mobile Robot Based on Dynamic Programming Algorithm," in *2nd International Conference on Electronic & Mechanical Engineering and Information Technology*, 2012.

[16] H. Choset, K. Lynch, S. Hutchinson, G. Kantor, W. Burgard, I. Kavraki and S. Thrun, Principals of Robot Motion: Theory, algorithms and implementations, MIT Press, 2005.

[17] D. H. Kim, G. Hoang, M. J. Bae, J. W. Kim, S. M. Yoon, T. K. Yeo, H. Sup and S. B. Kim, "Path Tracking Control Coverage of a Mining Robot Based on Exhaustive Path Planning with Exact Cell Decomposition," in *International Conference on Control, Automation and Systems*, 2014.

[18] H. Choset and P. Pignon, "Coverage Path Planning: The Boustrophedon Cellular Decomposition," in *Field and Service Robotics*, Springer, 1998, pp. 203-209.

[19] E. U. Acar, H. Choset, A. A. Rizzi, P. N. Atkar and D. Hull, "Morse Decomposition for coverage tasks.," *International Journal of Robotics Research,* vol. 21, no. 4, pp. 331-334, 2002.

[20] E. U. Acar, H. Choset and J. Y. Lee, "Sensor-Based Coverage with Extended Range Detectors," *IEEE Transactions on Robotics,* vol. 22, no. 1, pp. 189-198, 2006.

[21] B. Lau, C. Sprunk and W. Burgard, "Efficient grid-based spatial representations for robot navigation in dynamic environments," *Robotics and Autonomous Systems,* vol. 61, pp. 1116-1130, 2013.

[22] A. Yazici, G. Kirlik, O. Parlaktuna and A. Sipahioglu, "A Dynamic Path Planning Approach for Multirobot Sensor-Based Coverage Considering Energy Constraints," *IEEE Transactions on Cybernetics,* vol. 44, no. 3, pp. 305-314, 2014.

[23] B. Dugarjav, S. G. Lee, D. Kim, J. H. Kim and N. Y. Chong, "Scan Matching Online Cell Decomposition for Coverage Path Planning in an Unknown Environment," *International Journal of Precision Engineering and Manufacturing,* vol. 14, no. 9, pp. 1551-1558, September 2013.

[24] K. Yang, S. K. Gan and S. Sukkrarich, "An Efficient Path Planning and Control Algorithm for RUAV's in Unknown and Cluttered Environments," *Journal of Intelligent & Robotic Systems,* vol. 57, no. 1-4, pp. 101-122, 2010.

[25] S. M. LaValle, Planning Algorithms, Cambridge University Press, 2006.

[26] S. M. Lavelle, October 1998. [Online]. Available: http://msl.cs.uiuc.edu/~lavalle/papers/Lav98c.pdf.

[27] K. Yang, S. Moon, S. Yoo, J. Kang, N. L. Doh, H. B. Kim and S. Joo, "Spline-Based RRT Path Planner for Non-Holonomic," *J Intell Robot Syst,* vol. 73, pp. 763-782, 2014.

[28] S. Karaman and E. Frazzoli, "Sampling-based algorithms for optimal motion planning," *International Journal of Robotics Research,* vol. 30, no. 7, p. 846–894, 2011.

[29] Y. Gabriely and E. Rimon, "Spiral-STC : an online coverage algorithm of grid environments by a mobile robot," in *International conference on Robotics and Automation*, 2002.

[30] P. M. Hsu, C. L. Lin and M. Y. Yang, "On the Complete Coverage Path Planning for Mobile Robots," *Journal of Intelligent & Robotic Systems,* vol. 74, no. 3-4, pp. 945-963, June 2014.

[31] K. S. Senthilkumar and K. K. Bharadwaj, "Spanning Tree Based Terrain Coverage by Multi Robots in an Unknown Environment," in *India Conference, INDICON, Annual IEEE*, 2008.

[32] K. S. Senthilkumar and K. K. Bharadwaj, "Multi-robot exploration and terrain coverage in an unknown environment," *Robotics and Autonomous Systems,* vol. 60, pp. 123-132, 2012.

[33] S. Dasgupta, C. H. Papadimitriou and U. V. Vazirani, Algorithms, 1 ed., McGraw-Hill Education, 2006.

[34] G. Zuo, P. Zhang and J. Qiao, "Path Planning Algorithm Based on Sub-Region for Agricultural Robot," in *2nd International Asia Conference on Informatics in Control, Automation and Robotics*, 2010.

[35] M. Weiss-Cohen, K. I. ORT Braude Coll., I. Sirotin and E. Rave, "Lawn Mowing System for Known Areas," in *International Conference on Computational Intelligence for Modelling Control & Automation*, 2008.

[36] P. S. Paratama, J. W. Kim, K. H. Kim, S. M. Yoon, T. K. Yeu, S. Hong, S. J. Oh and S. B. Kim, "Path Planning Algorithm to Minimize an Overlapped Path and Turning Number for an Underwater Mining Robot," in *15th International Conference on Control, Automation and Systems*, Busan, Korea, 2015.

[37] E. Dijkstra, "A note on two problems in connexion with graphs," *Numerische Mathematik,* vol. 1, no. 1, pp. 269-271, 1958.

[38] P. E. Hart, N. J. Nilsson and B. Raphael, "A Formal Basis for the Heuristic Determination of Minimum Cost Paths," *IEEE Transactions on Systems Science and Cybernetics,* vol. 4, no. 2, pp. 100-107, 1968.

[39] P. Chand and D. E. Carnegie, "A two-tiered global path planning strategy for limited memory mobile robots," *Robotics and Autonomous Systems,* vol. 60, pp. 309-321, 2011.

[40] M. Eichhorn, "A New Concept for an Obstacle Avoidance System for the AUV "SLOCUM Glider" Operation under Ice," in *Oceans*, Bremen, Germany, 2009.

[41] M. Eichhorn, "Optimal routing strategies for autonomous underwatre vehicles in time-varying environment," *Robotics and Autonomous Systems, In press.*

[42] A. Stentz, "Optimal and Efficient Path Planning for Partially-Known Environments," in *Proceedings IEEE Conference on Robotics and Automation*, 1994.

[43] M. Dakulovic, S. Horvatic and I. Petrovic, "Complete Coverage D* Algorithm for Path Planning of a Floor-Cleaning Mobile Robot," in *18th International Federation of Automatic Control (IFAC), World Congress*, 2011.

[44] A. Nash, K. Daniel, S. Koenig and A. Felner, "Theta*: Any angle Path Planning on Grids," *Journal of Artificial Intelligence Research,* vol. 39, no. 1, pp. 533-579 , 2010.

[45] H. H. Viet, S. Y. Choi and T. C. Chung, "An online complete coverage approach for a team of robots in unknown environments," in *13th International Conference on Control, Automation and Systems*, Gwangju, Korea, 2013.

[46] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein, Introduction to Algorithms, 3rd Edition. ed., MIT Press, 2009.

[47] A. E. Eiben and J. E. Smith, Introduction to Evolutionary Computing, 2nd ed., Springer, 2007.

[48] I. Chaari, A. Koubaa, S. Trigui, H. Bennaceur, A. Ammar and K. Al-Shalfan, "SmartPATH: An Efficient Hybrid ACO-GA algorithm for solving the Global Path Planning Problem of Mobile Robots," *International Journal of Advanced Robotic Sytems, In press,* 2015.

[49] P. A. Jimenez, B. Shirinzadeh, A. Nicholson and G. Alici, "Optimal Area Covering using Genetic Algorithms," in *2007 IEEE/ASME international conference on Advanced intelligent mechatronics*, 2007.

[50] K. S. Senthilkumar and K. K. Bharadwaj, "An Efficient Global Optimization Approach to Multi Robot Path Exploration Problem Using Hybrid Genetic Algorithm," in *4th International Conference on Information and Automation for Sustainability*, Colombo, 2008.

[51] A. I. Hameed, D. Bochtis and C. A. Sorensen, "An Optimized Field Coverage Path Planning Approach for Navigation of Agricultural Robots in Fields Involving Obstacles," *Internation Journal of Advanced Robotic Systems,* vol. 10, pp. 1-9, 2013.

[52] W. M. Chong, C. L. Goh and Y. T. Bau, "A Practical Framework for Cleaning Robots," in *Sixth International Conference on Bio-Inspired Computing: Theories and Applications*, 2011.

[53] M. Kapanoglu, M. Alikalfa, M. Ozkan, A. Yazici and O. Parlaktuna, "A pattern-based genetic algorithm for multi- robot coverage path planning minimizing completion time," *Journal of Intelligent Manufacturing,* vol. 23, p. 1035–1045, 2012.

[54] I. A. Hameed, "Intelligent Coverage Path Planning for Agricultural Robots and Autonomous Machines on Three-Dimensional Terrain," *Journal of Intelligent and Robotic System,* vol. 47, pp. 965-983, 2014.

[55] M. Dorigo, *Ant Colony Optimization,* Cambridge University, Massachusetts: MIT Press, 1992.

[56] K. Zhou, A. L. Jensen, C. G. Sorensen, P. Busato and D. D. Bothtis, "Agricultural operations planning in fields with multiple obstacle areas," *Computers and Electronins in Agriculture,* vol. 109, pp. 12-22, 2014.

[57] Z. Chibin, W. Xingsong and D. Yong, "Complete Coverage Path Planning Based on Ant Colony Algorithm," in *15th International Conference on Mechatronics and Machine Vision in Practice* , Auckland , New Zealand, 2008.

[58] J. Kennedy and R. C. Eberhart, "Particle swarm optimization," in *Proceedings of the IEEE International Conference on Neural Networks*, 1995.

[59] G. Pessin, D. O. Sales, M. A. Dias, R. I. Klaser, D. F. Wolf, J. Ueyama, F. S. Osoria and P. A. Vargas, "Swarm intelligence and the quest to solve a garbage and recycling collection problem," *Soft Computing,* vol. 17, no. 12, pp. 2311-2325, 2013.

[60] M. A. H. Akhand, S. Akter, S. S. Rahman and M. M. Hafizur Rahman, "Particle Swarm Optimization with Partial Search to Solve Traveling Salesman Problem," in *International Conference on Computer and Communication Engineering*, 2012.

[61] N. Rokbani, A. Abraham and A. M. Alimi, "Fuzzy Ant Supervised by PSO and Simplified Ant Supervised PSO Applied to TSP," in *International Conference on Hybrid Intelligent Systems*, 2013.

[62] G. Farin, Curves and Surfaces for CAGD A Practical Guide, 5 ed., Morgan-Kaufman, 2002.

[63] A. Xu, C. Viriyasuthe and I. Rekleitis, "Efficient complete coverage of a known arbitrary environment with applications to aerial operations," *Autonomous Robots,* vol. 36, pp. 365-381, 2014.

[64] I. A. Hameed, D. D. Bochtis and C. G. Sorensen, "Driving Angle and Track Sequence Optimization for Operational Path Planning using Genetic Algorithms," *Applied Engineering in Agriculture, ASABE,* vol. 27, no. 6, pp. 1077-1086, 2011.

[65] D. D. Bochtis and S. G. Vougiouskas, "Minimizing the non-working distance travelled by machines operating in a headland field pattern," *Biosystems Engineering,* vol. 101, pp. 1-12, Aug. 2008.

[66] M. Spekken and S. D. Bruin, "Optimized routing on agriculture field by minimizing maneuvering and servicing time," *Precision Agriculture,* vol. 14, pp. 224-244, 2013.

[67] L. E. Dubin, "On curves of minimal length with constraint on average curvature, and with prescribed initial and terminal positions and tangents," *American Journal of Mathematics,* vol. 79, no. 3, pp. 497-516, 1957.

[68] J. Backman, P. Piirainen and T. Oksanen, "Smooth turning path generation for agricultural vehicles in headlands," *BioSystems Engineering,* vol. 139, pp. 76-86, August 2015.

[69] X. Yu, T. A. Roppel and J. Y. Hung, "An Optimization Approach for Planning Robotic Field Coverage," in *41st Annual Conference of the IEEE Inductrial Electronics Society*, Yokohama, 2015.

[70] C. Cariou, R. Lenain, B. Thuilot and P. Martinet, "Autonomous maneuver of a farm behicle with a trailed implement: motion planner and lateral-longitudinal controllers," in *IEEE International Conference on Robotics and Automation*, Anchorage, Alaska, USA, 2010.

[71] D. Sabelhaus, F. Roben and L. P. M. Helligen, "Using continuous-curvature paths to generate feasible headland turn manoeuvres," *Biosystems Engineering,* vol. 116, pp. 399-409, 2013.

[72] S. Surve, N. M. Singh and V. K. Lande, "CCPA: A fast coverage algorithm," in *International Conference on Computational Intelligence and Multimedia Applications*, 2007.

[73] A. Janchiv, D. Batsaikhan, B. Kim, W. G. Lee and S. G. Lee, "Time-Efficient and Complete Coverage Path Planning based on Flow Networks for Multi-Robots," *International Journal of Control, Automation and Systems,* vol. 2, no. 11, pp. 369-376, 2013.

[74] J. Valento, J. D. Cerro, A. Barrientos and D. Sanz, "Aerial Coverage Optimization in Precision Agriculture Management : A Musical Harmony Inspired Approach," *Computer and Electronics in Agriculture,* no. 99, pp. 153-159, 2013.